

# Application of Adaptive Kalman Filter in Dynamic Tuned Gyroscope Identification

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## Abstract

The accuracy of the dynamic tuned gyroscope parameters is crucial to its stability. In order to obtain accurate values of the actual parameters of the dynamic tuned gyroscope and avoid the impact of the inability to accurately establish the gyro noise model on the gyro parameter values during the identification process, the paper applies the adaptive Kalman filter algorithm to the parameter identification of dynamic tuned gyroscope. The paper explains the basic principles of adaptive Kalman filtering for system identification and deduces the formula. It also derives the adaptive Kalman filter identification model and the recursive least squares identification model of the dynamic tuned gyroscope based on the dynamic equation of the dynamic tuned gyroscope. Under the condition of unknown identification noise, the identification effects of the two algorithms were compared by changing the noise variance setting values in the two identification algorithms. Experimental results show that under the condition of unknown noise model or accurate value, the adaptive Kalman filter identification algorithm has better identification results and higher accuracy than the recursive least squares method.

## Keywords

Adaptive Kalman Filter; System Identification; Dynamically Tuned Gyroscope; Recursive Least Squares.

## 1. Introduction

As a classic mechanical rotor gyroscope with comprehensive cost-effectiveness, dynamic tuned gyroscope are still widely used in engineering practice due to their advantages such as mature technology, small size, moderate accuracy, and low cost[1]. If the real key parameters of dynamic tuned gyroscope products can be accurately identified and evaluated, it will be of great significance for improving the batch production qualification rate and quality stability of dynamic tuned gyroscope and developing high-performance gyro servo drive systems. Due to the complex structure of the dynamic tuned gyroscope itself and the dynamic changes in the noise of the identification environment, the noise model cannot be accurately established during the parameter identification process.

In order to solve the above problems, this paper proposes to use the adaptive Kalman filter identification algorithm to identify the parameters of the dynamic tuned gyroscope. The adaptive Kalman filter can dynamically adjust the model parameters and noise covariance by introducing an adaptive mechanism[2][3], which can simultaneously adjust the state quantity. Estimation is performed while estimating the parameters of the noise through measured values[4][5]. Using adaptive Kalman filter in dynamic tuned gyroscope identification can solve the problem of identifying gyro parameters under unknown noise model and improve accuracy.

## 2. Adaptive Kalman Filter Identification Algorithm

The difference equation of the identification system with single input and single output[6][7] is:

$$y(k) = -\sum_{i=1}^{n_a} a_i y(k-i) + \sum_{j=1}^{n_b} b_j u(k-j) + v(k) \tag{1}$$

Let the state vector  $X_k$  in the adaptive Kalman filter be equal to the identification parameter  $a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b}$  in the identification difference equation to solve the parameter value. Convert the dynamic difference equation describing the identification system into the corresponding state space equation form. The number of state vectors is the sum of the numbers of a and b that require identification parameters, that is  $n_a + n_b$ . In actual engineering applications, because the system parameters change extremely slowly, the dynamic behavior of the system remains relatively stable in a short period of time, so  $a(k)$  to  $a(k-1)$  can be regarded as almost unchanged, that is, the state transition matrix  $\Phi_{k/k-1}$  can be regarded as a unit matrix  $E$ , and the identification iteration The operational error of the process is  $w(k-1)$ . Although the error is small, it cannot be ignored. Then the state equation of the adaptive Kalman filter identification algorithm can be obtained as:

$$\theta_k = \theta_{k-1} + W_{k-1} \tag{2}$$

The difference equation and the measurement equation of the Kalman filter are both equations used to describe the current observation. When  $X_k = \theta_k$  and  $H_k = \Phi_k$ , the difference equation and the measurement equation are equivalent. Then the state space model of the adaptive Kalman filter identification algorithm can be obtained as:

$$\begin{cases} \theta_k = \theta_{k-1} + W_{k-1} \\ y(k) = \Phi_k \theta_k + v(k) \end{cases} \tag{3}$$

It can be seen that in the state space model of adaptive Kalman filter identification,  $\theta_k$  is the parameter to be identified,  $W_{k-1}$  is the iterative noise generated by the identification parameter  $x$ ,  $y(k)$  is the kth measurement output,  $v(k)$  is the kth identification environmental noise,  $\Phi_k$  is the combined vector of the first  $n_a$  measurement outputs of the kth time and the first  $n_b$  excitation signals of the kth time.

Since the identification environment is complex,  $v(k)$  is unknown. For the adaptive estimation of the noise variance  $R_k$  of the identification environment, the identification noise prediction error is:

$$\tilde{y}(k) = y(k) - \hat{y}(k) = \Phi_k \theta_k + v(k) - \Phi_k \hat{\theta}_{k/k-1} = \Phi_k \tilde{\theta}_{k/k-1} + v(k) \tag{4}$$

Calculating the variance of the noise prediction error, we can get:

$$E[\tilde{y}(k)^2] = \Phi_k P_{k/k-1} \Phi_k^T + e(k) \tag{5}$$

Using the exponential vanishing memory weighted average method, the equation for identifying the noise variance  $e(k)$  can be obtained as:

$$e(k) = (1 - \beta_k)e(k-1) + \beta_k((y(k) - \Phi_k \theta_k)^2 - \Phi_k P_{k/k-1} \Phi_k^T) \quad (6)$$

### 3. DTG Modeling and Modeling

When the dynamic tuned gyroscope is working, there is a relative deflection angle between the rotation axis of the gyro rotor and the drive axis, which causes torsional elastic deformation of the torsion bar and generates a positive elastic moment. At the same time, the balance ring undergoes torsional motion and generates a negative elastic moment to compensate for the positive elastic moment. This "dynamic tuning" effect jointly maintains the inertial stability of the gyro's rotation axis[8][9]. When designing a system identification experiment, the dynamic tuned gyroscope is fixed on a stationary experimental bench. At this time, the gyro shell is stationary relative to the inertial space. Under this condition, the simplified dynamic tuned gyroscope dynamic equation is:

$$\begin{cases} J\ddot{\beta} + \delta\dot{\beta} + \Delta k\beta + H\dot{\alpha} + \lambda\alpha = M_x \\ J\ddot{\alpha} + \delta\dot{\alpha} + \Delta k\alpha - H\dot{\beta} - \lambda\beta = M_y \end{cases} \quad (7)$$

In formula (7), the parameters of  $J$ ,  $\delta$ ,  $\Delta k$ ,  $H$ ,  $\lambda$  are the key identification objects in this article.

The moment of inertia  $J$  is the resistance of the gyroscope to changing angular velocity when it rotates around its own axis. It is crucial to the stability and tuning performance of the gyroscope. At the same time, by precisely controlling the moment of inertia, the sensitivity and response speed of the gyroscope can be tuned. A larger moment of inertia can make it more difficult for the gyroscope to change its rotation state, providing better stability and accuracy.

The damping coefficient  $\delta$  refers to the characteristics of the damping force experienced by the gyroscope when it rotates. The function of the damping coefficient is to reduce the oscillation and overshoot when the gyroscope rotates to improve the stability and accuracy of the gyroscope. Damping prevents the top from spinning indefinitely by absorbing the energy generated when the top rotates;

The residual elastic coefficient  $\Delta k$  is the residual elasticity existing in the gyro system. During the rotation of the gyro, even if the non-ideality of the gyro has been minimized during the design and manufacturing process, there is still an elastic effect. The residual elasticity coefficient is very important to the performance and accuracy of the gyroscope. It can cause the gyroscope to have undesirable deflection or deformation as it rotates, thus affecting the accuracy of its measurements.

Orthogonal damping elastic coefficient  $\lambda$  refers to the damping parameter related to the orthogonal vibration mode in gyro motion. Orthogonal damping is usually used to describe the mutual coupling and damping effects of gyros in multi-axis motion;

Angular momentum  $H$  is a physical quantity that describes the rotational motion of a gyro. It is determined by the rotational speed and moment of inertia of the gyro. The role of angular momentum is to measure and record the rotational motion of the gyro, thereby helping to determine the direction, attitude and rotational speed of the object.

The identification model is simplified after performing pull transformation on equation (7), setting the servo driving torque of the y-axis to 0, and only applying the servo driving torque to

the x-axis. When a torque is applied along the x-axis, the gyroscope will move along the vertical axis y-axis. The precession rotation of the gyro is generated in the direction, and the rigid body rotation is generated on the parallel x-axis. The precession swing angle of the y-axis is much larger than the rotation swing angle of the x-axis, so equation (8) is obtained as the main identification formula of the dynamic tuned gyroscope. According to Eq. (8) Construct the continuous transfer function as shown in equation (9):

$$\alpha(s) = \frac{Hs + \lambda}{(Js^2 + \delta s + \Delta k)^2 + (Hs + \lambda)^2} M_x(s) \quad (8)$$

$$G(s) = \frac{d_1 s + d_0}{s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0} \quad (9)$$

The continuous domain transfer function cannot be directly used in the identification model because the identification process processes digital signals, so the continuous domain transfer function needs to be converted into a discrete domain transfer function. This article chooses the bilinear transformation method to analyze the continuous transfer function of the dynamic tuned gyroscope. For conversion, the bilinear transformation method can better maintain the frequency response of the system than the zero-order keeper method. It can make the system more stable than the differential transformation method. It is easier to calculate and more capable than the zero-pole matching method. It has engineering application value. Compared with the impulse response invariant method, it has lower requirements on excitation signals and is the best choice for dynamic tuned gyroscope model transformation. The discrete domain transfer function after converting equation (9) through bilinear transformation is:

$$G(z) = \frac{y(z)}{u(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}} \quad (10)$$

## 4. Experiment

### 4.1. Identify Experimental Prior Conditions

This paper chooses multi-harmonic difference signals as the excitation signals to identify the parameters of the dynamic tuned gyroscope. The multi-harmonic differential signal is a periodic signal. It is a signal obtained by modulating and mixing multiple sinusoidal signals of different frequencies. Therefore, it has rich frequency diversity. The multi-harmonic differential signal can also excite the system under test. The response at different frequencies improves the frequency characteristics of the identification system. Because it also contains multiple frequency components, it can cover a wide frequency range and help capture the dynamic behavior of the system in multiple frequency ranges. Harmonic phase signals can also be adjusted to meet different identification needs by adjusting the frequency and amplitude of the signal.

When selecting the iterative operation cycle, the dynamic characteristics of the system need to be taken into consideration. For most continuous systems, it is best to choose a shorter iterative operation cycle. However, in actual dynamic tuned gyroscope identification experiments, the shorter iterative operation cycle is. The more difficult it will be for actual operation, so in order to meet the above two requirements, the iterative operation cycle cannot be set too long or too short. Therefore, the iterative operation period was set to 0.0015s in the experiment. Since the

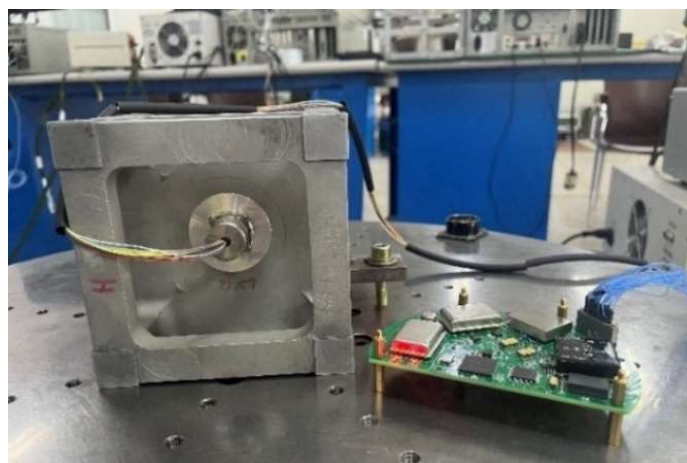
swing angles of the x-axis and y-axis rotors of the dynamic tuned gyroscope are small, both between  $\pm 0.6^\circ$ , the amplitude of the designed multi-harmonic differential signal is not easy to be too high, and the amplitude of each harmonic is set to 0.002V. If the fundamental frequency is set to  $f_1 = 0.2\text{Hz}$ , then the second harmonic frequency  $f_2 = 0.4\text{Hz}$ , the third harmonic frequency  $f_3 = 0.6\text{Hz}$ , and the fourth harmonic frequency  $f_4 = 0.8\text{Hz}$ , the phases of the above harmonics are randomly distributed in  $0 \sim 2\pi$ .

The identification time of the experiment should be neither too long nor too short. Although a longer identification time can ensure sufficient data sampling and sufficient information to identify the dynamic characteristics of the system, it will increase the experimental and labor costs and reduce the value of engineering applications. A shorter identification time will result in a lack of sufficient information and make the identification less accurate. Therefore, choosing a moderate identification time can better balance time and cost when experimental resources are sufficient. This article chooses 210s as the gyro identification experiment time.

#### 4.2. Composition and Principle

The TZ-A138 dynamic tuned gyroscope assembly consists of an NT12 dynamic tuned gyroscope and a TSY-59 dynamic tuned gyroscope digital servo driver. It measures the rotation angular rate of the gyro carrier around two axes during flight and has a sensitive carrier biaxial angle. It is a function of speed and outputs a DC voltage signal proportional to it. Its accuracy directly affects the stability accuracy and measurement accuracy of the gyro carrier. The NT12 dynamic tuned gyroscope and the TSY-59 dynamic tuned gyroscope digital servo driver are installed separately, as shown in Fig 1.

When the carrier drives the gyro assembly to rotate relative to the inertial space, due to the fixed axis of the gyro rotor, the sensor outputs a voltage signal proportional to the rotation angle. After preamplification, phase-sensitive demodulation, servo correction, and power amplification, this signal becomes a DC current signal proportional to the angular rate. It is input to the torque device and acts on the gyro rotor, causing the rotor to precess rapidly and tightly. By tracking the rotation of the gyroscope and measuring the current value of the torque device, the angular velocity of the carrier can be measured.



**Fig.1** Components of dynamic tuned gyroscope

#### 4.3. Overview of Parameter Identification Experimental Hardware System

The working principle of the dynamic tuned gyroscope parameter identification experimental hardware system is as follows: the dynamic tuned gyroscope digital servo driver DSP data operation and processing unit performs real-time calculation of the multi-harmonic differential signal mathematical function to generate an excitation digital signal for gyro parameter

identification. The digital signal After DA conversion and power amplification, it becomes a gyro torque current applied to the gyro torque device. This torque current generates a torque that drives the gyro to precess. Under the action of this torque, the gyro rotor swings, and the gyro rotor swing angle information is converted into a digital signal by the signal conditioning circuit and AD conversion circuit. The DSP data operation and processing unit collects and processes the digital signal. The DSP data operation and processing unit packages the self-generated multi-harmonic differential phase excitation signal data and the collected gyro rotor swing angle signal data into RS422 serial port data frames, and uploads these data to the test computer through the RS422 serial port. The test computer receives and records these test data, and uses these data to complete the offline solution of dynamic tuned gyroscope parameter identification.

**4.4. Identification Scheme Design**

The experiment was completed by the TZ-A138 dynamic tuned gyroscope assembly. The dynamic tuned gyroscope parameter identification experiment was designed as follows: the NT-12 dynamic tuned gyroscope was fixed on a stationary experimental bench, and the gyro axis was controlled by the TSY-59 dynamic tuned gyroscope servo system. The torque device applies a driving current that changes according to the set mathematical characteristics. The driving torque generated by this driving current is the excitation signal required for identification. At the same time, it records the change data of the gyro axis rotor swing angle, and combines the axis drive current data and the axis gyro swing angle data. As the input data of the identification calculation system, the required key parameters of the gyroscope are calculated through identification calculation.

The identification calculation process is as follows: input the processed driving current data and gyro swing angle data into the programs of the adaptive Kalman filter identification algorithm and the recursive least squares identification algorithm, obtain the value of the identification parameter aa, and substitute the parameters into the discrete transfer function In the model, the discrete transfer function model is converted into a continuous domain transfer function model using bilinear inverse transformation to obtain the value of bb, and the value of the gyro identification parameter cc is finally obtained through calculation.

**4.5. Experimental Data Processing and Analysis**

**Table 1.** Comparison of adaptive Kalman filtering and recursive least squares identification results

Discrete Identification Parameters	Standard Value	AKF Identification	RLS Identification
$a_1$	-1.02541	-0.99449	-1.28904
$a_2$	0.11089	0.09891	0.35879
$a_3$	-0.16896	-0.16158	-0.80378
$a_4$	0.069645	0.06719	0.37563
$b_1$	7.71257	7.69681	5.87696
$b_2$	40.13413	40.19744	35.58097
$b_3$	54.95663	55.12621	44.03567
$b_4$	37.14401	37.06248	20.53711

When the y-axis swing angle minus the initial offset and the torque current are input into the adaptive Kalman filter identification algorithm and the recursive least squares identification

algorithm, the noise variances in the two identification algorithms are set to be  $1 \times 10^{-6}$ ,  $1 \times 10^{-7}$ ,  $1 \times 10^{-8}$  and  $1 \times 10^{-9}$ , the average data of discrete identification parameter values is shown in Table 1, and the final average value of gyro identification parameters is shown in Table 2.

**Table 2.** Comparison of gyro parameter identification values

DTG Parameters	Experience Value	AKF Identification	RLS Identification	Improved accuracy
$H$	$5.744 \times 10^{-4}$	$5.7813 \times 10^{-4}$	$7.3032 \times 10^{-4}$	26.4%
$J$	$3.659 \times 10^{-7}$	$3.5576 \times 10^{-7}$	$1.8744 \times 10^{-7}$	46.1%
$\delta$	$5.5141 \times 10^{-4}$	$5.5199 \times 10^{-4}$	$7.0783 \times 10^{-4}$	26.38%
$\lambda$	$2.0742 \times 10^{-1}$	$2.0782 \times 10^{-1}$	$2.0976 \times 10^{-1}$	41.7%
$\Delta k$	$9.6972 \times 10^{-2}$	$9.7543 \times 10^{-2}$	$7.9492 \times 10^{-2}$	18.7%

It can be seen from Table 1 and Table 2 that when the experimental environment noise is unknown, different noise variance R values are set in the two identification algorithms. The adaptive Kalman filter identification algorithm has higher accuracy, while the recursive least squares algorithm has higher accuracy due to the noise variance. The setting does not meet the actual identification noise variance, resulting in low identification accuracy. As can be seen from Table 2, when the noise variances are set to  $1 \times 10^{-6}$ ,  $1 \times 10^{-7}$ ,  $1 \times 10^{-8}$  and  $1 \times 10^{-9}$ , the overall accuracy of the adaptive Kalman filter identification algorithm is improved by 31.85% compared to the recursive least squares algorithm.

## 5. Summary

This paper focuses on the problem of inability to accurately model the dynamic tuned gyroscope identification due to the noisy identification environment or the complex gyro structure, and applies the adaptive Kalman filter algorithm to the dynamic tuned gyroscope parameter identification. Experimental results show that compared with the traditional recursive least squares identification method, the adaptive Kalman filter identification algorithm can still identify accurately when the noise is unknown and the set noise variance is inaccurate, effectively solving the problem of inaccuracies in establishing the noise model in dynamic tuned gyroscope identification. Accuracy leads to the problem of low identification accuracy.

## References

- [1] Cai Yao, Si Yuhui, Wang Yuzhuo, etc. Research on axial static balance of small flexible gyro without adjustment mechanism [J]. Aviation Precision Manufacturing Technology, 2023, 59(04): 34-39.
- [2] Hosseini S M, Ranjbar Noei A, Sadati Rostami S J. Integrated navigation system (INS/auxiliary sensor) based on adaptive robust Kalman filter with partial measurements[J]. Transactions of the Institute of Measurement and Control, 2023, 45(2): 316-330.
- [3] Zhang L, Shaoping W, Selezneva M S, et al. A new adaptive Kalman filter for navigation systems of carrier-based aircraft[J]. Chinese Journal of aeronautics, 2022, 35(1): 416-425.
- [4] Wei Tong, Guo Rui. Application of adaptive Kalman filtering in brushless DC motor system identification [J]. Optical Precision Engineering, 2012, 20(10): 2308-2314.
- [5] Huang B, Wang J, Zhang J, et al. Variational Bayesian Adaptive Kalman Filter for Integrated Navigation with Unknown Process Noise Covariance[C]//2022 2nd International Conference on Consumer Electronics and Computer Engineering (ICCECE). IEEE, 2022: 436-443.

- [6] Wu Xingxing. Aeroengine modeling and application based on system identification method [J]. Science and Technology Innovation and Application, 2022, 12(23): 18-21.
- [7] Manel C, Mohamed A. Fourth-order cumulants based-least squares methods for fractional Multiple-Input-Single-Output Errors-In-Variables system identification [J]. Fractional Calculus and Applied Analysis, 2023, 26(4): 1868-1893.
- [8] Wang Yingjian, Wang Zheng, Fang Zheng. Structural optimization of dynamically tuned gyroscope flexible joints based on BP-GA algorithm [J]. Modern Manufacturing Technology and Equipment, 2023, 59(04): 135-137.
- [9] Wu Chen; Guo Baolin; Zhang Peixin; Li Wei; Wu Zhan. Optimal design of flexible joints for micro power tuning gyros [J]. Aerospace Manufacturing Technology, 2023, (03): 17-22.