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Event-Triggered Differentially Private Consensus with The Noise Parameter

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Abstract

The event-triggered differentially private consensus problem with the noise parameter is investigated by giving the detailed analysis about convergence, accuracy, privacy and optimal variance. An additional noise parameter can adjust the size of the added noise flexibly. The controller with absolute information can reduce the calculation load. Moreover, we use the rigorous definition of infimum to prove optimal variance, which is a new method. Numerical results are provided to illustrate the feasibility of the proposed mechanism and the correctness of the theoretical results.

Keywords

Differentially private consensus, the noise parameter, distributed event-triggered mechanism, measurement error.

1. Introduction

With the development of society, more and more attention has been paid to privacy protection. Due to the rigorous formulation and proven security properties, the mechanism of differential privacy has been extensively studied when it was first introduced in [1]. In the application [2] of differential privacy to privacy protection, the data to be processed can be mainly divided into two categories: numeric and non-numeric. Laplacian mechanism and Gaussian mechanism are generally adopted for numeric queries, while exponential mechanism is adopted for non-numeric queries. Besides, Laplacian mechanism and exponential mechanismcan preserve ϵ-differential privacy, while Gaussian mechanism can preserve (ϵ, δ)-differential privacy. For ϵ-differential privacy, the parameter ϵ represents the degree of the privacy protection, and a smaller value of ϵ can guarantee a stronger privacy. For (ϵ, δ)-differential privacy, the parameter ϵ also represents the privacy degree and δ represents the probability of violating the privacy. When we set smaller values of ϵ and δ, we can get a higher privacy. Note that ϵ-differential privacy usually can provide a stronger privacy than (ϵ, δ)-differential privacy. For all kinds of mechanisms, the literature [3] gives the general properties of differential privacy and the basic conditions of the noise which guarantees differential privacy. Further, Huang [4] adds an independent and exponentially decaying Laplacian noise to the consensus process who firstly combines differential privacy with the average consensus. For preserving the privacy of initial states, Manitara [5] only gives the condition to make the initial state of one agent can be exactly recognized by the others, while Mo [6] provides a quantitative condition to estimate it perfectly. What's more, Nozari [7], [8] show that any differentially private algorithm can't achieve exact average consensus but achieve average consensus in expectation. To avoid real-time communication and controller updates frequently, Hermann [9] compares the properties of event-triggered and time-triggered distributed real-time systems, which shows that each agent i sends messages to neighbors if it satisfies the event-triggered condition, while the activities are initiated periodically at predetermined points in a time-triggered system. We can find that the event-triggered mechanism is more flexible than the time-triggered mechanism. On the other hand, the concept of measurement error [10] is firstly introduced into the eventtriggered mechanism. More references about event-triggered can be seen in [11], [12], [13], [14]. Recently, Gao [15] firstly adopts the event-triggered scheme to the differentially private consensus algorithms

$$\begin{aligned} &\theta_i(t+1)=\theta_i(t)+hu_i(t)+s_i\eta_i(t),\\ &u_i(t)=\sum_{j\in N_i} w_{ij}(x_j(t^j_{k_j})-x_i(t^i_{k_i})),\\ &x_i(t^i_{k_i})=\theta_i(t^i_{k_i})+s_i\eta_i(t^i_{k_i}). \end{aligned} \tag{1}$$

where $i\in\{1,2,\cdots,N\}$, $u_i(t)$ is the controller and Ni is the neighbor set of agent i. $\theta_i(t)$ is the internal state of agent i, $x_i(t)$ is the pre-transmitted messages of agent i sending to its neighbors, $h>0$ is the step size and $s_i>0$ is the noise parameter of agent i. $\eta_i(t)$ is the random noise, which obeys the Laplacian distribution Lap(ciqi(t)) with $q_i(t)\in(0,1)$ and the positive constant c_i. $\theta_i(t^i_{k_i})$ and $\eta_i(t^i_{k_i})$ represent the internal state and the random noise of agent i at its event time, respectively. $x_i(t^i_{k_i})$ represents the transmitted messages of agent i at the last event times.

The algorithm (1) with the additional noise parameter makes measurement error automatically reset to zero when an event is triggered and it has better universality. Specifically, if all agents no longer need privacy preservation, it will degenerate to a based event-triggered consensus algorithm by making all $s_i=0,\forall i\in\{1,2,3,\cdots,N\}$. Here, it's worth pointing out that $u_i(t)$ uses the relative information.

More recently, Wang [16] gives a new algorithm to use the absolute information. It can be expressed as

$$\begin{aligned} &\theta_i(t+1)=\alpha_i\theta_i(t)+\beta_i\sum_{j\in N_i} a_{ij}x_j(t^j_{k_j}),\\ &x_i(t)=\theta_i(t)+\eta_i(t). \end{aligned} \tag{2}$$

where $0<\alpha_i,\beta_i<1$. Comparing (2) with (1), it is not difficult to find that the algorithm (2) can reduce the computation load between neighbors. The system (2) without the noise parameter means that the noise must be added, and the size of the noise cannot be adjusted. However, noise may not be needed in a particular environment.

In this paper, we design a new algorithm by adding si into the system (2)

$$\begin{aligned} &\theta_i(t+1)=\alpha_i\theta_i(t)+\beta_i\sum_{j\in N_i} a_{ij}x_j(t^j_{k_j}),\\ &x_i(t)=\theta_i(t)+s_i\eta_i(t). \end{aligned} \tag{3}$$

which can be seen to combine the advantages of algorithm (1) and algorithm (2). Because we design the controller with absolute information, the execution efficiency is improved compared with algorithm (1). Moreover, the addition of noise parameter also makes our system more universal. Under the premise of obeying the Laplacian distribution, we can control the size of added noise.

To better investigate this algorithm (3), the probability density function, probability, expectation and variance of a random variable X are denoted by $f(X)$, $P(X)$, $E(X)$ and $V(X)$. $V[X]=E[X^2]-E[X]E[X]$. $X\sim Lap(b)$ means that a zero mean random variable X obeys the Laplacian distribution with $V[X]=2b^2$. The probability density function is

$f(x;b)=\frac{1}{2b}e^{-\frac{|x|}{b}}$. Let $G=\{V,E,A\}$ be an undirected and connected communication graph, in which $V=\{1,2,\cdots,N\}$ is the node set, $E\subseteq V\times V$ is the edge set and $A=(a_{ij})_{N\times N}$ is the adjacency matrix of G. The neighbor set of agent i is defined as $N_i=\{j\in V\,|\,(i,j)\in E\}$. We don't consider self-loop in this paper, which means $a_{ii}=0$. The Laplacian matrix $L=(l_{ij})_{N\times N}$ associated with the adjacency matrix A is defined by $l_{ij}=-a_{ij}$, $i\neq j$, $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$. We order the eigenvalues of the Laplacian matrix in the increasing order as $0=\lambda_1(L)\leq\lambda_2(L)\leq\cdots\leq\lambda_N(L)$.

Next, we propose the event-triggered condition to determine the next event time of the algorithm (3) as follows

$$t_{k_i+1}^{i}=\inf\{t:t>t_{k_i}^{i},\ f_i(t)=0\},\tag{4}$$

where $e_i(t)$ is the measure error with $e_i(t)=x_i(t_{k_i}^{i})-x_i(t)$ and $0<\sigma_i<1$. We choose "=" in the event-triggered condition, which is similar to Xiong [17]. According to $e_i(t)=x_i(t_{k_i}^{i})-x_i(t)$ we can know that $e_j(t)=x_j(t_{k_j}^{j})-x_j(t)$. Substituting $x_j(t_{k_j}^{j})=x_j(t)+e_j(t)$ into the first equation of (3) gives

$$\theta_i(t+1)=\alpha_i\theta_i(t)+\beta_i\sum_{j\in N_i}a_{ij}(x_j(t)+e_j(t)),\tag{5}$$

thus,

$$\theta_i(t+1)=\alpha_i\theta_i(t)+\beta_i\sum_{j\in N_i}a_{ij}x_j(t)+\beta_i\sum_{j\in N_i}a_{ij}e_j(t).\tag{6}$$

Next,we can get the vector form for this system as follows

$$\begin{aligned}&\theta(t+1)=\alpha\theta(t)+\beta Ax(t)+\beta Ae(t),\\&x(t)=\theta(t)+S\eta(t),\end{aligned}\tag{7}$$

where $\alpha=diag\{\alpha_1,\alpha_2,\cdots\alpha_N\}$, $\beta=diag\{s_1,s_2,\cdots s_N\}$, $S=diag\{s_1,s_2,\cdots s_N\}$, $\eta(t)=(\eta_1(t),\eta_2(t),\cdots,\eta_N(t))^T$, $\theta(t)=(\theta_1(t),\theta_2(t),\cdots,\theta_N(t))^T$ and $x(t)=(x_1(t),x_2(t),\cdots x_N(t))^T$. Note that here we assume $A=(a_{ij})_{N\times N}$ is a doubly stochastic square matrix and $0\leq a_{ij}\leq 1$, which means that

$$\begin{cases}\lambda_1(A)=1,|\lambda_i(A)<1|,i=2,3,\cdots,N,\\ A\mathbf{1}=\mathbf{1},\end{cases}\tag{8}$$

where $\mathbf{1}=(1,1,\cdots,1)^T$.

Remark 1. Due to the algorithm (3) has the noise parameter s_i, we can adjust the noise level at any time. Furthermore, Our event-triggered condition (4) is more efficient that only depends on local information and local parameters of agent i, which is different from [15] and [16].

The remaining of this paper is organized as follows. Firstly, we give convergence analysis, accuracy analysis, differential privacy analysis and optimal variance discussion in section 2. Followed in section 3, we provide some simulations to support our results. Finally, we conclude this paper in section 4.

2. Main Results

In this section, we will explain the rationality of the proposed algorithm (3) from four aspects, such as convergence, accuracy, privacy and optimal variance.

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2.1. Convergence Analysis

In the subsection, we will analyze the convergence of multiagent system from two aspects. On the one hand, it is proved that the agents can achieve mean square convergence by constructing a Lyapunov function, and a sufficient condition for algorithm (3) to reach mean square asymptotic convergence is proposed. On the other hand, the convergence rate of the multi-agent system is calculated according to the definition, and how the parameters in algorithm (3) affect the convergence

rate of the agents are shown.

Definition 1. (Mean Square Consensus [4]): The agents are said to reach consensus in mean square if for any agents $i, j \in \{1, 2, \cdots, N\}$, $\lim_{t\to\infty} E[(\theta_i(t) - \theta_j(t))^2] = 0$ holds.

Theorem 1. The mechanism described in (3) achieves the mean square asymptotic convergence for all agents if $\max[\alpha_i + \beta_i(1-\hat{\sigma}_i)^{-1}] < 1$ holds.

Proof: Firstly, we construct a Lyapunov function $P(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}(\theta_i(t) - \theta_j(t))^2$ which can be rewritten as $P(t) = \frac{1}{2}\theta^T(t)L\theta(t)$. According to the event-triggered condition (4), we have

$$e(t) = \sigma(x(t) + e(t)), \tag{9}$$

where $\sigma = diag\{\hat{\sigma}_1, \hat{\sigma}_2, ..., \hat{\sigma}_N\}$, with $\hat{\sigma}_i = \pm\sigma_i$. We can rewrite (9) as

$$e(t) = (I-\sigma)^{-1}\sigma x(t). \tag{10}$$

Substituting (10) into (7), we can get

$$\begin{aligned} &\theta(t+1) = \alpha\theta(t) + (\beta A + \beta A(I-\sigma)^{-1}\sigma)x(t), \\ &x(t) = \theta(t) + S\eta(t). \end{aligned} \tag{11}$$

Thus, we have

$$\begin{aligned} P(t+1) &= \frac{1}{2}\theta^T(t+1)L\theta(t+1) \\ &= \frac{1}{2}\left\{\alpha\theta(t) + \left[\beta A + \beta A(I-\sigma)^{-1}\sigma\right]x(t)\right\}^T L \\ &\quad\times\left\{\alpha\theta(t) + \left[\beta A + \beta A(I-\sigma)^{-1}\sigma\right]x(t)\right\} \\ &= \frac{1}{2}\left\{\theta^T(t)\alpha^T + x^T(t)\left(A^T\beta^T + \sigma^T\left[\left(I-\sigma\right)^{-1}\right]^T A^T\beta^T\right)\right\}L \\ &\quad\times\left\{\alpha\theta(t) + \left[\beta A + \beta A\left(I-\sigma\right)^{-1}\sigma\right]x(t)\right\} \\ &= \frac{1}{2}\left\{\theta^T(t)\alpha + \left[\theta^T(t) + \eta^T(t)S\right]\left[A\beta + \sigma\left(I-\sigma\right)^{-1}A\beta\right]\right\}L \\ &\quad\times\left\{\alpha\theta(t) + \left[\beta A + \beta A\left(I-\sigma\right)^{-1}\sigma\right]\left[\theta(t) + S\eta(t)\right]\right\}. \end{aligned} \tag{12}$$

In order to make the complicated derivation more simple in the description , we make a simple substitution for the parameters. Let $H = (I-\sigma)^{-1}A\beta$. Then $A\beta = (I-\sigma)H = H - \sigma H$. $H^T = ((\mathrm{I}-\sigma)^{-1}\mathrm{A}\beta)^T = \beta A(\mathrm{I}-\sigma)^{-1}$, so $\beta A = H^T(\mathrm{I}-\sigma) = H^T - H^T\sigma$. Therefore, (12) can be rewritten as

$$\begin{aligned}P(t+1) &= \frac{1}{2}\left\{\theta^T(t)\alpha + \left[\theta^T(t)+\eta^T(t)S\right]\left(H-\sigma H+\sigma H\right)\right\}L \\ &\quad \times\left\{\alpha\theta(t)+\left(H^T-H^T\sigma+H^T\sigma\right)\left(\theta(t)+S\eta(t)\right)\right\} \\ &= \frac{1}{2}\left\{\theta^T(t)\alpha+[\theta^T(t)+\eta^T(t)S]H\right\}L\left\{\alpha\theta(t)+H^T[\theta(t)+S\eta(t)]\right\} \\ &= \frac{1}{2}\left[\theta^T(t)\alpha+\theta^T(t)H+\eta^T(t)SH\right]L\left[\alpha\theta(t)+H^T\theta(t)+H^TS\eta(t)\right] \\ &= \frac{1}{2}\theta^T(t)\alpha L\alpha\theta(t)+\frac{1}{2}\theta^T(t)\alpha LH^T\theta(t)+\frac{1}{2}\theta^T(t)\alpha LH^TS\eta(t) \\ &\quad +\frac{1}{2}\theta^T(t)HL\alpha\theta(t)+\frac{1}{2}\theta^T(t)HLH^T\theta(t)+\frac{1}{2}\theta^T(t)HLH^TS\eta(t) \\ &\quad +\frac{1}{2}\eta^T(t)SHL\alpha\theta(t)+\frac{1}{2}\eta^T(t)SHLH^T\theta(t)+\frac{1}{2}\eta^T(t)SHLH^TS\eta(t).\end{aligned} \tag{13}$$

For $\forall i,j\in\{1,2,\cdots,N\}$, it follows from the literature [15] that $\theta_i(t)$ and $\eta_i(t)$ are independent of each other, $\eta_i(t)$ and $\eta_j(t)$ are also independent of each other for any $\forall i\neq j$. Moreover, $E[\eta_i(t)]=0$ holds.

$$\begin{aligned}E[P(t+1)] &= E[\frac{1}{2}\theta^T(t)\alpha L\alpha\theta(t)]+E[\frac{1}{2}\theta^T(t)\alpha LH^T\theta(t)]+E[\frac{1}{2}\theta^T(t)\alpha LH^TS\eta(t)] \\ &\quad +E[\frac{1}{2}\theta^T(t)HL\alpha\theta(t)]+E[\frac{1}{2}\theta^T(t)HLH^T\theta(t)]+E[\frac{1}{2}\theta^T(t)HLH^TS\eta(t)] \\ &\quad +E[\frac{1}{2}\eta^T(t)SHL\alpha\theta(t)]+E[\frac{1}{2}\eta^T(t)SHLH^T\theta(t)]+E[\frac{1}{2}\eta^T(t)SHLH^TS\eta(t)] \\ &= E[\frac{1}{2}\theta^T(t)\alpha L\alpha\theta(t)]+E[\frac{1}{2}\theta^T(t)\alpha LH^T\theta(t)]+0 \\ &\quad +E[\frac{1}{2}\theta^T(t)HL\alpha\theta(t)]+E[\frac{1}{2}\theta^T(t)HLH^T\theta(t)]+0 \\ &\quad +0+0+E[\frac{1}{2}\eta^T(t)SHLH^TS\eta(t)] \\ &\leq \max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2 E[P(t)] \\ &\quad +\frac{1}{2}\max\left\{\left[s_i\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2\right\}\lambda_N(L)E\left[\eta^T(t)\eta(t)\right],\end{aligned} \tag{14}$$

where $\lambda_N(L)$ is the maximum eigenvalue of the Laplacian matrix. If we can guarantee that $\max[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}]<1$, the first term of (14) will converge to 0 as $t\to\infty$. Since $E[\eta_i^2(t)]=V[\eta_i(t)]=2c_i^2q_i^{2t}$ and $0<q_i<1$, so we have $E[\eta_i^2(t)]\to 0$ as $t\to\infty$. Furthermore, the second term of (14) converges to 0 as $t\to\infty$. Therefore, the algorithm (3) achieves the mean square asymptotic convergence as $t\to\infty$.

The convergence rate is a very important scale for the convergence of multi-agent system, we can calculate the convergence rate to understand the internal relationship between the parameters of multi-agent system and convergence. The defination of convergence rate is given as follows.

Definition 2. (Convergence Rate) [6]: We define the exponential mean-square convergence rate as

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$$Q=\lim_{t\to\infty}\left(\sup\frac{E\theta^T(t)\theta(t)}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}. \tag{15}$$

When $0<Q<1$, the system will reach mean square convergence, when $Q>1$, the system can't reach mean square convergence.

Now, according the definition 2, we compute the convergence rate Q.

$$\begin{aligned}
\lim_{t\to\infty}\left(\frac{E\theta^T(t)\theta(t)}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}} &\le \lim_{t\to\infty}\left(\frac{\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2 E[\theta^T(t-1)\theta(t-1)]}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}\\
&=\lim_{t\to\infty}\left(\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2\frac{E[\theta^T(t-1)\theta(t-1)]}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}\\
&\le\lim_{t\to\infty}\left(\left\{\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2\right\}^2\frac{E[\theta^T(t-2)\theta(t-2)]}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}\\
&\cdots\cdots\\
&\le\lim_{t\to\infty}\left(\left\{\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2\right\}^t\frac{[\theta^T(0)\theta(0)]}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}\\
&\le\lim_{t\to\infty}\left(\left\{\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2\right\}^t\right)^{\frac{1}{t}}\\
&=\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2,
\end{aligned} \tag{16}$$

Therefore, we can get the convergence rate

$$Q=\lim_{t\to\infty}\left(\sup\frac{E\theta^T(t)\theta(t)}{\theta^T(0)\theta(0)}\right)^{\frac{1}{t}}=\max\left[\alpha_i+\beta_i(1-\hat{\sigma}_i)^{-1}\right]^2. \tag{17}$$

2.2. Accuracy Analysis

In the subsection, we prove that the weighted average of an agent will converge to the weighted average of the agent's initial state information by combining the relevant theoretical knowledge of probability theory, and we propose a sufficient condition for an agent to achieve (p, r)-accuracy.

Definition 3. (Accuracy) [4]: For any initial $\theta(0)$, a randomized mechanism is said to achieve (p,r)-accuracy if every execution converges to a state with probability at least $1-p$.

Theorem 2. Set $\gamma_i=\frac{1}{1-\alpha_i}$, if $\beta_i=(1-\alpha_i)(1-\hat{\sigma}_i)$, the algorithm (3) achieves $\left(p,\sqrt{\frac{2}{p}\sum_{i=1}^{N}\frac{s_i^2c_i^2d}{1-q_i^2}}\right)$-accuracy for any $p\in(0,1)$.

Proof: It follows from (11) that

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$$\begin{aligned}\theta(t+1) &= \alpha\theta(t) + [\beta A + \beta A(I-\sigma)^{-1}\sigma](\theta(t) + S\eta(t)) \\ &= \alpha\theta(t) + \beta A[I + (I-\sigma)^{-1}\sigma]\big(\theta(t) + S\eta(t)\big) \\ &= \alpha\theta(t) + \beta A[(I-\sigma)^{-1}(I-\sigma) + (I-\sigma)^{-1}\sigma]\big(\theta(t) + S\eta(t)\big) \\ &= \alpha\theta(t) + \beta A[(I-\sigma)^{-1}(I-\sigma+\sigma)]\big(\theta(t) + S\eta(t)\big) \\ &= \alpha\theta(t) + \beta A(I-\sigma)^{-1}\big(\theta(t) + S\eta(t)\big).\end{aligned} \tag{18}$$

which can be written in the distributed form as

$$\theta_i(t+1) = \alpha_i\theta_i(t) + \beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}\theta_j(t) + \beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}s_j\eta_j(t). \tag{19}$$

Let $\bar{\theta}(t) = \dfrac{\sum_{i=1}^{N}\gamma_i\theta_i(t)}{\sum_{i=1}^{N}\gamma_i}$ and multiply by γ_i on the both sides of (19)

$$\begin{aligned}\gamma_i\theta_i(t+1) &= \gamma_i\alpha_i\theta_i(t) + \gamma_i\beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}\theta_j(t) \\ &\quad + \gamma_i\beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \gamma_i\theta_i(t) - \gamma_i(1-\alpha_i)\theta_i(t) \\ &\quad + \gamma_i\beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}\theta_j(t) + \gamma_i\beta_i(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \gamma_i\theta_i(t) - \frac{1}{1-\alpha_i}(1-\alpha_i)\theta_i(t) \\ &\quad + \frac{1}{1-\alpha_i}(1-\alpha_i)(1-\hat{\sigma}_i)(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}\theta_j(t) \\ &\quad + \frac{1}{1-\alpha_i}(1-\hat{\sigma}_i)(1-\alpha_i)(1-\hat{\sigma}_i)^{-1}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \gamma_i\theta_i(t) - \theta_i(t) + \sum_{j\in N_i} a_{ij}\theta_j(t) + \sum_{j\in N_i} a_{ij}s_j\eta_j(t).\end{aligned} \tag{20}$$

Note that the self-loop isn't considered in this paper, which means $a_{ii} = 0$, so $\sum_{j\in N_i} a_{ij}\theta_j(t)$ equals to $\sum_{j=1}^{N} a_{ij}\theta_j(t)$. Furthermore,

$$\begin{aligned}\sum_{i=1}^{N}\gamma_i\theta_i(t+1) &= \sum_{i=1}^{N}\gamma_i\theta_i(t) - \sum_{i=1}^{N}\theta_i(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\theta_j(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \sum_{i=1}^{N}\gamma_i\theta_i(t) - \sum_{i=1}^{N}\theta_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\theta_j(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \sum_{i=1}^{N}\gamma_i\theta_i(t) - \sum_{i=1}^{N}\theta_i(t) + \sum_{j=1}^{N}(\sum_{i=1}^{N} a_{ij})\theta_j(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \sum_{i=1}^{N}\gamma_i\theta_i(t) - \sum_{i=1}^{N}\theta_i(t) + \sum_{j=1}^{N}\theta_j(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}s_j\eta_j(t) \\ &= \sum_{i=1}^{N}\gamma_i\theta_i(t) + \sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}s_j\eta_j(t).\end{aligned} \tag{21}$$

Multiply by $\frac{1}{\sum_{i=1}^{N}\gamma_i}$ on both sides of (19), we have

$$\begin{aligned}\bar{\theta}(t+1) &= \bar{\theta}(t)+\tilde{w}(t)\\ &= \bar{\theta}(t-1)+\tilde{w}(t-1)+\tilde{w}(t)\\ &= \bar{\theta}(t-2)+\tilde{w}(t-2)+\tilde{w}(t-1)+\tilde{w}(t)\\ &\cdots\cdots\\ &= \bar{\theta}(0)+\sum_{k=0}^{t}\tilde{w}(k),\end{aligned} \tag{22}$$

where $\tilde{w}(t)=\frac{\sum_{i=1}^{N}\sum_{j\in N_i}a_{ij}s_j\eta_j(t)}{\sum_{i=1}^{N}\gamma_i}$. Thus, we have

$$E[\bar{\theta}(t+1)]=E[\bar{\theta}(0)]. \tag{23}$$

Note the independent of $\eta_i(t)$ for all agents, we have

$$\begin{aligned}V[\tilde{w}(t)] &= V\left[\frac{\sum_{i=1}^{N}\sum_{j\in N_i}a_{ij}s_j\eta_j(t)}{\sum_{i=1}^{N}\gamma_i}\right]\\ &= \frac{V[\sum_{i=1}^{N}\sum_{j\in N_i}a_{ij}s_j\eta_j(t)]}{(\sum_{i=1}^{N}\gamma_i)^2}\\ &= \frac{V[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}s_j\eta_j(t)]}{(\sum_{i=1}^{N}\gamma_i)^2}\\ &= \frac{V[\sum_{j=1}^{N}(\sum_{i=1}^{N}a_{ij})s_j\eta_j(t)]}{(\sum_{i=1}^{N}\gamma_i)^2}\\ &= \frac{\sum_{i=1}^{N}V[s_i\eta_i(t)]}{(\sum_{i=1}^{N}\gamma_i)^2}\\ &= \frac{1}{(\sum_{i=1}^{N}\gamma_i)^2}\sum_{i=1}^{N}2s_i^2{c_i}^2{q_i}^{2t}\\ &= 2\frac{1}{(\sum_{i=1}^{N}\gamma_i)^2}\sum_{i=1}^{N}s_i^2{c_i}^2{q_i}^{2t}\\ &= 2d\sum_{i=1}^{N}s_i^2{c_i}^2{q_i}^{2t}.\end{aligned} \tag{24}$$

We combine (24) with the independent of $\tilde{w}(t)$, which implies that

$$\begin{aligned}V[\sum_{k=0}^{t}\tilde{w}(k)] &\le V[\sum_{k=0}^{\infty}\tilde{w}(k)]\\ &= 2d\sum_{k=0}^{\infty}\sum_{i=1}^{N}s_i^2{c_i}^2{q_i}^{2k}\\ &= \sum_{i=1}^{N}\frac{2{s_i}^2{c_i}^2d}{1-{q_i}^2},\end{aligned} \tag{25}$$

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where $d=\frac{1}{(\sum_{i=1}^{N}\gamma_i)^2}$. By Chebyshev's inequality for any $t\ge 0$,

$$P(\left|\bar{\theta}(t)-\bar{\theta}(0)\right|\le r)\ge 1-\frac{V\left[\sum_{k=0}^{t}\tilde{w}(k)\right]}{r^2}, \tag{26}$$

Now, we choose $r=\sqrt{\frac{2}{p}\sum_{i=0}^{N}\frac{s_i^2c_i^2d}{1-q_i^2}}$, so $P(\left|\bar{\theta}(t)-\bar{\theta}(0)\right|\le r)\ge 1-p$, which means that the algorithm (3) can achievethe $\left(p,\sqrt{\frac{2}{p}\sum_{i=1}^{N}\frac{s_i^2c_i^2d}{1-q_i^2}}\right)$-accuracy.

2.3. Differential Privacy Analysis

In the subsection, we show that the observation sequence of any given two sets of δ- adjacent data sets will be indistinguishable after the action of algorithm (3), and then it is proved that the algorithm (3) can guarantee the ϵ-difference privacy of agent's initial state by combining probability theory and triangle inequality.

Definition 4. (Differential Privacy) [4]: For any given pair of δ-adjacent initial states $\theta^{(1)}(0)$, $\theta^{(2)}(0)$, the system is said to preserve ϵ-differential privacy, if

$$P\left\{\mathrm{Alg}(\theta^{(1)}(t))\in\Theta\right\}\le e^{\varepsilon\delta}P\left\{Alg(\theta^{(2)}(t))\in\Theta\right\}$$

holds, where $Alg(\cdot)$ represents the execution of the algorithm (3), and Θ denotes the state domain of global execution.

Given $\delta>0$, a pair of δ-adjacent initial states $\theta^{(1)}(0)$, $\theta^{(2)}(0)$ is considered as follows

$$\left|\theta_i^{(2)}(0)-\theta_i^{(1)}(0)\right|\le\begin{cases}\delta, & if\quad i=i_0,\\ 0, & if\quad i\ne i_0.\end{cases} \tag{27}$$

Two groups of random noise $\eta_i^{(1)}(t)$ and $\eta_i^{(2)}(t)$ are designed as follows

$$\eta_i^{(2)}(t)=\begin{cases}\eta_i^{(1)}(t)+\frac{\delta}{s_i}\alpha_i^t, & if\quad i=i_0,\\ \eta_i^{(1)}(t), & if\quad i\ne i_0.\end{cases} \tag{28}$$

Proposition 1. According to (27) and (28), for $t\in\{0,1,\cdots\cdots\}$, we have $x_i^{(2)}(t)=x_i^{(2)}(t)$ for $\forall i\in\{1,2,\cdots,N\}$ and

$$\theta_i^{(2)}(t)=\begin{cases}\theta_i^{(1)}(t)-\delta\alpha_i^t, & if\quad i=i_0,\\ \theta_i^{(1)}(t), & if\quad i\ne i_0.\end{cases} \tag{29}$$

Proof: Take $t=0, i=i_0$, we have

$$\begin{aligned} x_{i_0}^{(2)}(0)&=\theta_{i_0}^{(2)}(0)+s_{i_0}\eta_{i_0}^{(2)}(0)\\ &=\theta_{i_0}^{(1)}(0)-\delta+s_{i_0}(\eta_{i_0}^{(1)}(t)+\frac{\delta}{s_{i_0}})\\ &=\theta_{i_0}^{(1)}(0)-\delta+s_{i_0}\eta_{i_0}^{(1)}(t)+\delta\\ &=\theta_{i_0}^{(1)}(0)+s_{i_0}\eta_{i_0}^{(1)}(t)\\ &=x_{i_0}^{(1)}(0). \end{aligned} \tag{30}$$

If $t=0, i\neq i_0$, we have

$$\begin{aligned} x_i^{(2)}(0) &= \theta_i^{(2)}(0)+s_i\eta_i^{(2)}(0) \\ &= \theta_i^{(1)}(0)+s_i\eta_i^{(1)}(0) \\ &= x_i^{(1)}(0). \end{aligned} \tag{31}$$

Thus, when $t=0, \forall i,$ we have $x_i^{(2)}(t)=x_i^{(1)}(t)$ for any agents.

Now, for $t=t'\neq 0$, we assume that $x_i^{(2)}(t')=x_i^{(1)}(t')$, $\forall i$, and

$$\theta_i^{(2)}(t')=\begin{cases}\theta_i^{(1)}(t')-\delta\alpha_i^{t'}, & if \quad i=i_0, \\ \theta_i^{(1)}(t'), & if \quad i\neq i_0. \end{cases} \tag{32}$$

Next, take $t=t'+1, i=i_0$, we have

$$\begin{aligned} \theta_{i_0}^{(2)}(t'+1)-\theta_{i_0}^{(1)}(t'+1) &= \alpha_{i_0}\theta_{i_0}^{(2)}(t')-\alpha_{i_0}\theta_{i_0}^{(1)}(t') \\ &= \alpha_{i_0}(\theta_{i_0}^{(1)}(t')-\delta\alpha_{i_0}^{t'})-\alpha_{i_0}\theta_{i_0}^{(1)}(t') \\ &= -\alpha_{i_0}\delta\alpha_{i_0}^{t'} \\ &= -\delta\alpha_{i_0}^{t'+1}, \end{aligned} \tag{33}$$

thus,

$$\theta_{i_0}^{(2)}(t'+1)=\theta_{i_0}^{(1)}(t'+1)-\delta\alpha_{i_0}^{t'+1}, \tag{34}$$

and we have

$$\begin{aligned} x_{i_0}^{(2)}(t'+1) &= \theta_{i_0}^{(2)}(t'+1)+s_{i_0}\eta_{i_0}^{(2)}(t'+1) \\ &= \theta_{i_0}^{(1)}(t'+1)-\delta\alpha_{i_0}^{t'+1}+s_{i_0}(\eta_{i_0}^{(1)}(t'+1)+\frac{\delta}{s_{i_0}}\alpha_{i_0}^{t'+1}) \\ &= \theta_{i_0}^{(1)}(t'+1)+s_{i_0}\eta_{i_0}^{(1)}(t'+1) \\ &= x_{i_0}^{(1)}(t'+1), \end{aligned} \tag{35}$$

Therefore, we have $x_i^{(2)}(t)=x_i^{(1)}(t)$ for ∀ i by using mathematical induction. In other words, the observation sequence of any given two sets of δ-adjacent data sets will be indistinguishable after the action of algorithm (3).

Theorem 3. For $q_i\in(\alpha_i,1)$, the mechanism guarantees ϵ-differential privacy with $\varepsilon_i=\frac{q_i}{s_ic_i(q_i-\alpha_i)}$ and $\varepsilon=\max_i \varepsilon_i, i\in\{1,2,\cdots,N\}$.

Proof: According to the Proposition 1 and the second equation of algorithm (3), we find that the privacy calculation of state $\theta_i^{(1)}(t)$ and $\theta_i^{(2)}(t)$ can be converted into the privacy calculation of noise $\eta_i^{(1)}(t)$ and $\eta_i^{(2)}(t)$, which obey Laplacian distribution. And, we have

$$P\left\{Alg(\eta(t)^{(\iota)})\in\Theta\right\}=\lim_{t\to\infty}\int_{R^{(\iota)}(t)} f_{N(t+1)}(\eta^{(\iota)}(t))d\eta^{(\iota)}(t), \tag{36}$$

where $Alg(\cdot)$ represents the execution of the algorithm (3), Θ denotes the state domain of global execution, $\iota=1,2$ $R^{(\iota)}(t)=\left\{Alg(\eta(t)^{(\iota)})\in\Theta\right\}$ and $f_{N(t+1)}$ is the $N(t+1)$-dimensional joint Laplacian probability distribution function given by

$$f_{N(t+1)}(\eta(t))=\prod_{i=1}^{N}\prod_{j=0}^{t}L(\eta_i(j);b_i(j)). \tag{37}$$

Based on (36) and (37), we can get

$$
\begin{aligned}
\frac{f_{N(t+1)}(\eta^{(1)}(t))}{f_{N(t+1)}(\eta^{(2)}(t))} &= \frac{\prod_{i=1}^{N}\prod_{j=0}^{t} L(\eta_i^{(1)}(j);b_i(j))}{\prod_{i=1}^{N}\prod_{j=0}^{t} L(\eta_i^{(2)}(j);b_i(j))} \\
&= \frac{\prod_{i=1}^{N}\prod_{j=0}^{t} L(\eta_i^{(1)}(j);b_i(j))}{\prod_{i=1}^{N}\prod_{j=0}^{t} L(\eta_i^{(1)}(j)+\Delta\eta_i(j);b_i(j))} \\
&= \frac{\prod_{j=0}^{t} L(\eta_{i_0}^{(1)}(j);b_{i_0}(j))}{\prod_{j=0}^{t} L(\eta_{i_0}^{(2)}(j);b_{i_0}(j))} \\
&= \frac{\prod_{j=0}^{t} \frac{1}{2b}\exp\left[-\frac{\left|\eta_{i_0}^{(1)}(j)\right|}{b_{i_0}(j)}\right]}{\prod_{j=0}^{t} \frac{1}{2b}\exp\left[-\frac{\left|\eta_{i_0}^{(1)}(j)\right|+\Delta\eta_{i_0}(j)}{b_{i_0}(j)}\right]} \\
&= \prod_{j=0}^{t}\exp\left[\frac{\left|\eta_{i_0}^{(1)}(j)+\Delta\eta_{i_0}(j)\right|-\left|\eta_{i_0}^{(1)}(j)\right|}{b_{i_0}(j)}\right] \\
&\le \prod_{j=0}^{t}\exp\left[\frac{\left|\eta_{i_0}^{(1)}(j)\right|+\left|\Delta\eta_{i_0}(j)\right|-\left|\eta_{i_0}^{(1)}(j)\right|}{b_{i_0}(j)}\right] \\
&= \prod_{j=0}^{t}\exp\left[\frac{\left|\Delta\eta_{i_0}(j)\right|}{b_{i_0}(j)}\right] \\
&= \exp\left[\sum_{j=0}^{t}\frac{\left|\Delta\eta_{i_0}(j)\right|}{b_{i_0}(j)}\right] \\
&= \exp\left[\sum_{j=0}^{t}\frac{\frac{\delta}{s_{i_0}}\alpha_{i_0}^{j}}{c_{i_0}q_{i_0}^{j}}\right] \\
&= \exp\left[\frac{\delta}{s_{i_0}c_{i_0}}\sum_{j=0}^{t}\left(\frac{\alpha_{i_0}}{q_{i_0}}\right)^{j}\right] \\
&= \exp\left[\frac{\delta}{s_{i_0}c_{i_0}}\frac{1-\left(\frac{\alpha_{i_0}}{q_{i_0}}\right)^{t+1}}{1-\left(\frac{\alpha_{i_0}}{q_{i_0}}\right)}\right].
\end{aligned}
\tag{38}
$$

According to (38) and $q_{i_0}\in(\alpha_{i_0},1)$, when $t\to\infty$, we have

$$
\frac{f_{N(t+1)}(\eta^{(1)}(t))}{f_{N(t+1)}(\eta^{(2)}(t))} \le \exp\left[\frac{q_{i_0}\delta}{s_{i_0}c_{i_0}(q_{i_0}-\alpha_{i_0})}\right]. \tag{39}
$$

Then intergrating the both sides of (39), we obtain the probability

$$
P\{Alg(\eta^{(1)}(t))\in\Theta\} \le \exp(\varepsilon_{i_0}\delta)P\left\{Alg(\eta^{(2)}(t))\in\Theta\right\}, \tag{40}
$$

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where $\varepsilon_{i_0} = \frac{q_{i_0}}{s_{i_0} c_{i_0} (q_{i_0} - \alpha_{i_0})}$, which means that (3) achieves ε_{i_0} -differential privacy. We need to notice that the agent i0 can be any agents. Consequently, the algorithm is ϵ-differential private with $\varepsilon = \max_i \varepsilon_i$, and a smaller value of ϵ can guarantee a stronger privacy.

2.4. Optimal Variance Discussion

According to the (25) and $\varepsilon_i = \frac{q_i}{s_i c_i (q_i - \alpha_i)}$,, we have an explicit privacy-accuracy trade-off between $V[\sum_{k=0}^{\infty} \tilde{w}(k)]$ and ε_i , where

$$V\left[\sum_{k=0}^{\infty} \tilde{w}(k)\right] = 2d\sum_{i=1}^{N} \frac{q_i^2}{\varepsilon_i^2 (1-q_i^2)(q_i - \alpha_i)^2}. \tag{41}$$

In the subsection, we consider as cost function of the variance of the agents' convergence point

$$J(\{\varepsilon_i, q_i, \alpha_i\}) = 2d\sum_{i=1}^{N} \frac{q_i^2}{\varepsilon_i^2 (1-q_i^2)(q_i - \alpha_i)^2}. \tag{42}$$

Next, we will optimize this trade-off.

Theorem 4. For the adjacent bound δ > 0 and the given privacy level ϵi, the optimal value of the variance of the agents' convergence point is

$$J^* = 2d\sum_{i=1}^{N} \frac{1}{\varepsilon_i^2}\left[\inf \frac{q_i^2}{(1-q_i^2)(q_i - \alpha_i)^2}\right] = 2d\sum_{i=1}^{N} \frac{1}{\varepsilon_i^2}, \tag{43}$$

where the privacy level ϵi can be fixed according to the agents' privacy requirements.

Proof: For $0 < \alpha_i < q_i < 1$, let

$$\psi(q_i, \alpha_i) = \frac{q_i^2}{(1-q_i^2)(q_i - \alpha_i)^2} \tag{44}$$

To prove (43), we first show that $\inf \psi(q_i, \alpha_i) = 1$ as Fig.8. It is sufficient to show that for any $\varepsilon > 0$, there exist α_0 and q_0 satisfing $0 < \alpha_0 < q_0 < 1$ such that

$$\psi(q_0, \alpha_0) < \varepsilon + 1. \tag{45}$$

To this end, for any $\varepsilon > 0$, let $k > k_0 = \frac{\sqrt{\varepsilon+1}}{\sqrt{\varepsilon+1} - 1}$, then $k_0 > 1$

$$\frac{k^2}{(k-1)^2 (\varepsilon + 1)} < 1. \tag{46}$$

Furthermore, for $0 < \alpha_0 < 1$, due to the arbitrary of ε, there is $ak > k_0$ such that $0 < k\alpha_0 < 1$. Now, let $q_0 = k\alpha_0$, then $0 < \alpha_0 < q_0 < 1$. Thus, we have

$$\psi(q_0, \alpha_0) < \frac{k^2}{(1 - k^2 \alpha^{*2})(k-1)^2} = \varepsilon + 1, \tag{47}$$

Note that $\alpha^* = \left[\frac{1}{k^2}\left(1 - \frac{k^2}{(\varepsilon+1)(k-1)^2}\right)\right]^{\frac{1}{2}}$, then it follows from (46) that $\alpha^* < 1$. So, for any $0 < \alpha_0 < \alpha^*$, a direct calculation from (46) and (47) shows that

$$\psi(q_0,\alpha_0) < \frac{k^2}{(1-k^2\alpha^{*2})(k-1)^2} = \varepsilon + 1, \tag{48}$$

which means that (45) holds and the proof is completed.

Remark 2. We give the optimal selection of some parameters to minimize the variance, and we prove it by using the definition of infimum, which is more rigorous and is different from the previous method.

3. Simulation Example

In this section, some numerical results are provided to illustrate the feasibility of the proposed algorithm (3) and the correctness of the theoretical results. The parameters are set $\alpha_i = 0.8$, $\beta_i = 0.02$, $\gamma_i = 5$, $\hat{\sigma}_i = 0.9$, $c_i = 0.2$, $q_i = 0.86$, where $i \in \{1,2,\cdots,N\}$. In this paper, we consider a undirected and connected network with six agents, the initial state information of agents are given as $\theta(0) = (0.04, 0.03, 0.02, -0.02, -0.03, -0.04)$. Then, the topology is shown in Fig.1, which is an octahedron designed to describe the state information transfer between six agents. The weight matrix of network and the Laplacian matrix are as follows

$$A = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.4 & 0.1 & 0.1 \\ 0.2 & 0 & 0.2 & 0.1 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.1 & 0.2 & 0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0 & 0.3 \\ 0.1 & 0.3 & 0.2 & 0.1 & 0.3 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & -0.2 & -0.2 & -0.4 & -0.1 & -0.1 \\ -0.2 & 1 & -0.2 & -0.1 & -0.2 & -0.3 \\ -0.2 & -0.2 & 1 & -0.2 & -0.2 & -0.2 \\ -0.4 & -0.1 & -0.2 & 1 & -0.2 & -0.1 \\ -0.1 & -0.2 & -0.2 & -0.2 & 1 & -0.3 \\ -0.1 & -0.3 & -0.2 & -0.1 & -0.3 & 1 \end{bmatrix}.$$

Fig.2, Fig.3 and Fig.4 show the state trajectory $\theta_i(t)$ of 6 agents with the different s_i. We find that the greater the noise parameters of the system, the greater the disturbance caused. When the noise added to the system is too large, although it can play a good role in protecting privacy, it may lead to data distortion. Therefore, it is very necessary to select the appropriate size of noise. The differential privacy mechanism adopted in this paper make the agent cannot achieve accurate convergence, because there is always random noise in the system to disturb the multi-agent system. However, we can find that there is a moment T, the trajectory of all agents' state information will approach infinitely to a random variable as $t > T$, which will randomly fall in the vicinity of the weighted average value of the agent's initial state information with limited disturbance variance. Since the random noise we added obeys the Laplacian distribution and is expected to be 0, that is to say, although the system cannot achieve exact convergence, it can achieve convergence to a constant in expectation, where the constant is the weighted average of the initial state information of the agents. Fig.5, Fig.6 and Fig.7 show the evolution of the function $P(t)$ with the different s_i. It is not difficult to find that $P(t) \to 0$ as $t \to \infty$, which means that the algorithm (3) achieves the mean square asymptotic convergence. Fig.8 shows that $\psi(q_i,\alpha_i)$ approaches its infimum 1 as $\alpha_i \to 0$, $q_i \to 0$.

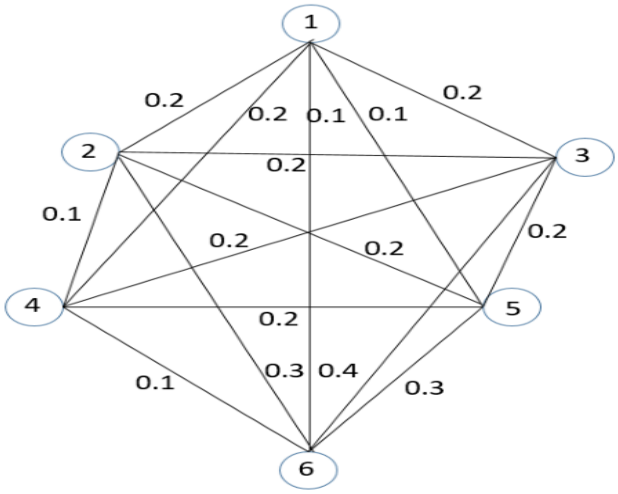

Fig. 1. The weighted interaction network with 6 agents.

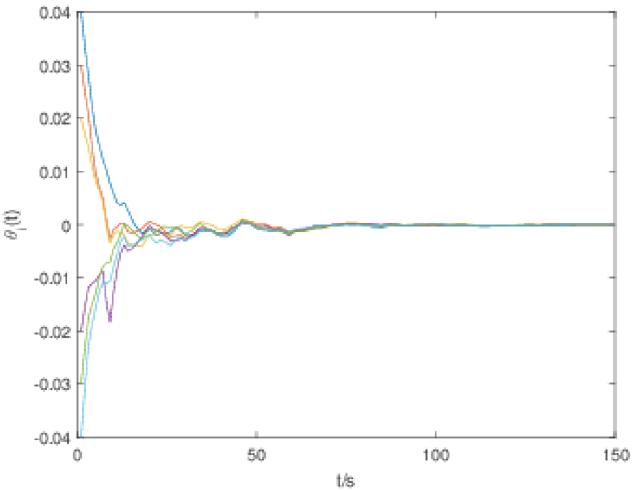

Fig. 2. State trajectory $\theta_i(t)$ of 6 agents with $s_i = 0.99$.

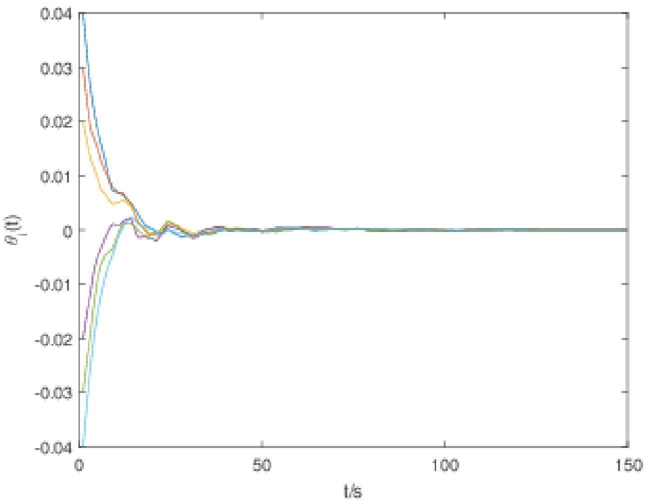

Fig. 3. State trajectory $\theta_i(t)$ of 6 agents with $s_i = 0.5$.

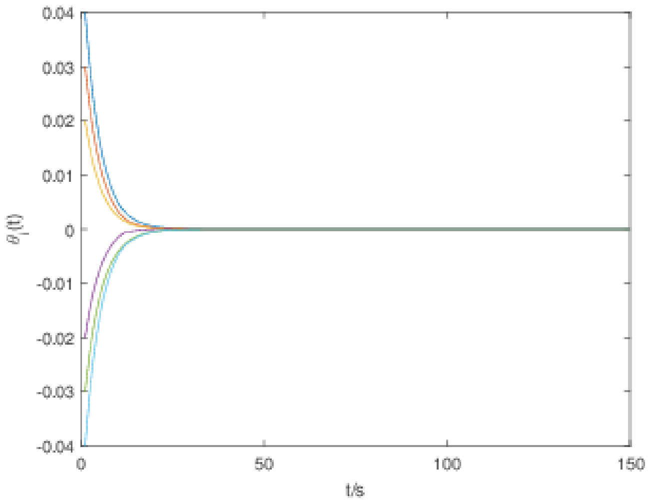

Fig. 4. State trajectory $\theta_i(t)$ of 6 agents with $s_i = 0$.

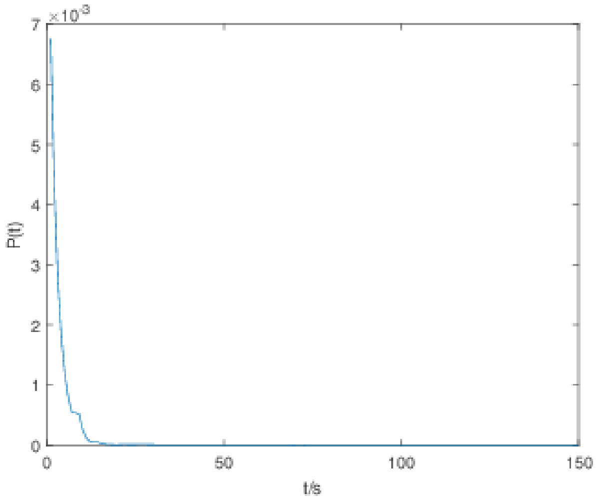

Fig. 5. Evolution of the function $P(t)$ with $s_i = 0.99$.

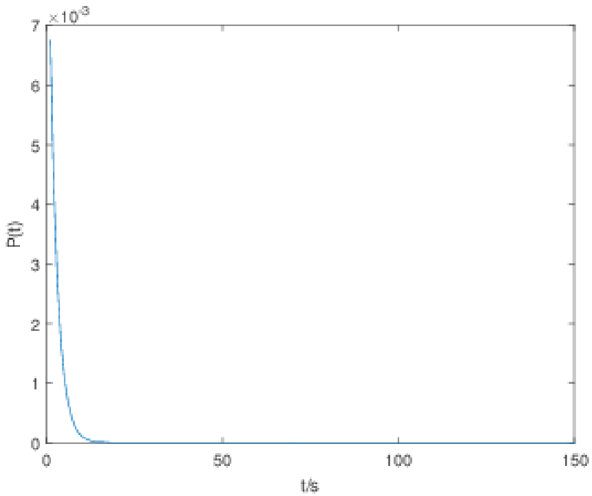

Fig. 6. Evolution of the function $P(t)$ with $s_i = 0.5$.

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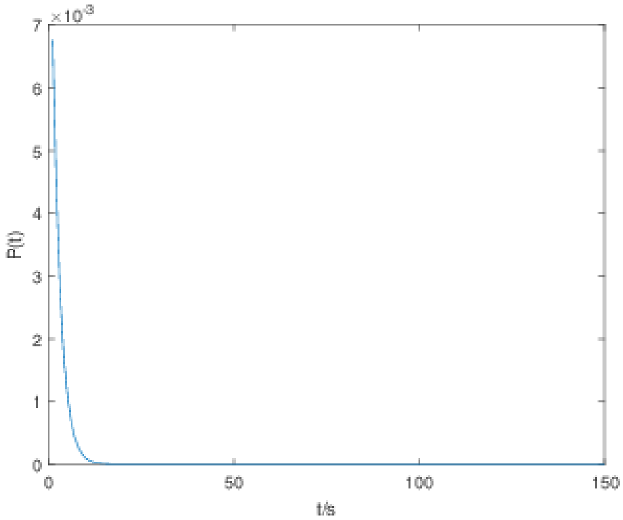

Fig. 7. Evolution of the function $P(t)$ with $s_i = 0$.

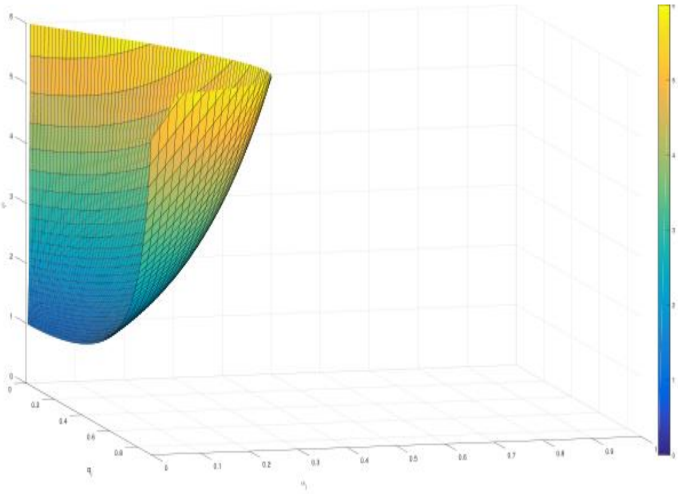

Fig. 8. The graph of the function $\psi(q_i, \alpha_i)$.

4. Conclusion

In this paper, we study the event-triggered differentially private consensus with the noise parameter. The new algorithm not only preserves the privacy of each agent' initial state but also improves the execution efficiency of network. We prove our algorithm can achieve the mean square asymptotic convergence by constructing a Lyapunov function. Futhermore, we combine knowledge of probability theory to give the accuracy analysis. What's more, according to the mathematical induction, we prove two observation sequences are indistinguishable when the initial states of agents are δ-adjacent and analize the differential privacy. Last but not least, we select the optimal parameters and prove it with the rigorous definition of infimum. Of course, the space for research is still very large, we can add the time-delay into system.

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