

# Design of Optimal Color Scheme for Opaque Products Based on Nonlinear Programming Model

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## Abstract

**This article focuses on how to achieve color matching for opaque products through computer methods, and constructs nonlinear programming models based on K-M single constant theory and CIELAB color difference calculation method to design the optimal formula for opaque products.**

## Keywords

**Opaque product color matching; Regression fitting; Nonlinear programming; Multi-objective programming.**

## 1. Introduction

In the rapid development of modern technological production, intelligent digital color matching is also accelerating the implementation of fast and efficient development strategies. The computer color matching system is a modern equipment that integrates a color measuring instrument, a computer, and a color matching software system. As an intelligent color matching system, it is widely used in the field of color matching in different industries. It can not only quickly match and measure colors, but also greatly improve work efficiency and reduce unnecessary waste of human, material, and financial resources. By introducing a computer color matching model, traditional inefficient manual color matching has gradually developed into more refined and efficient computer color matching, and has gradually become the mainstream color matching method. Implementing color matching for opaque products through computer methods can effectively solve the limitations of traditional manual color matching such as strong subjectivity and low efficiency, and can improve the appearance aesthetics of opaque products and enhance market competitiveness. Therefore, Studying how to use computer methods to match colors for opaque products has important practical significance.

## 2. Literature review

The method of using computers to predict dyeing formulas began in the 1930s. The K-M optical model and color additive mixing model are the foundation of computer color matching theory. In 1940, when Duncan studied paint color matching<sup>[1]</sup> and in 1942, when Saunderson studied plastic color<sup>[2]</sup>, they both assumed that the absorption coefficient  $K$  and scattering coefficient  $S$  of the color material were proportional to the concentration. After Allen introduced the scattering and absorption coefficients of the substrate, it ultimately evolved into a color additive mixing model. Jiang Pengfei systematically analyzed the K-M optical model in the article "Research on Computer Color Matching Theory and Algorithms"<sup>[3]</sup>, and provided a detailed overview and analysis of the relationship between K-M absorption coefficient  $K$ , scattering coefficient  $S$ , physical absorption coefficient, and scattering coefficient; Wang Linji

studied the conversion relationship of color parameters in different color spaces and the total color difference calculation method in CIELAB color space to calculate color difference in the article "Research on Blended Color Measurement Based on CIELAB Uniform Color Space and Clustering Algorithm"<sup>[4]</sup>. This article mainly establishes a model to solve problems by learning and utilizing the formulas and concepts in the provided references.

### 3. Problem exploration

Question 1: Calculate the relationship between K/S and concentration of red, yellow, and blue colorants at different wavelengths, and obtain their relationship formula and fitting coefficient.

Question 2: Given the R value of the target sample, based on the weighted table of spectral tristimulus values and the K/S basic database of colorants, establish an appropriate optimization model. When the color difference is less than 1, prepare 10 different formulas that are closest to the color difference of the target sample.

Question 3: Considering the two constraints of cost control and batch color matching, improve the color matching model and color match a 2kg base material. When the color difference is less than 1, find the 10 different formulas that are closest to the color difference between the target sample.

## 4. Establishment and Solution of the Model

### 4.1. Solution to Question 1

According to the K-M optical model, there is a certain relationship between the absorption coefficient K/scattering coefficient S of the colorant and the reflectivity R. Assuming the concentration of the colorant is C (in grams) and the K/S value is K/S (C) at a wavelength of, this article uses polynomial regression to fit and obtain the relationship expression between C and K/S at different concentrations and wavelengths.

Polynomial regression fitting is a common data fitting method and an improved linear regression model. Its basic idea is to use polynomial functions to fit the data, in order to find a curve that can best fit the data. To ensure the fitting effect, the fitting coefficients obtained from this question are all greater than or equal to 97%. According to the data distribution, first-order polynomial regression can be used to fit the data, and its expression is:

$$K/S((\lambda, C))=a*C+b$$

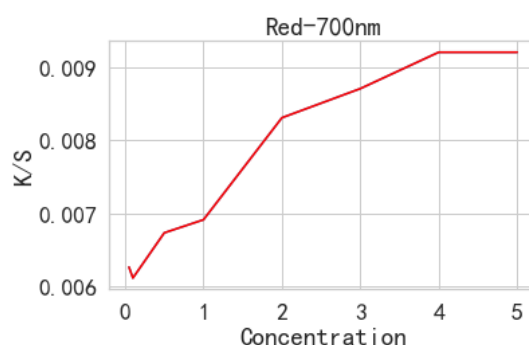


Figure 1 Line graph of C-K/S (700, C) under red colorant

When the colorant is red and the wavelength is 700nm, the C-K/S value line graph is shown in Figure 1. The relationship presented in the line graph is no longer suitable for first-order fitting, and it can be calculated that the fitting coefficient of the first-order fitting is 0.9258, indicating a lower fitting effect. Therefore, second-order fitting is used. The functional relationship and fitting coefficients of C-K/S values for three different colorants at different wavelengths are shown in Table 1:

Table 1 Functional Relationships and Fit Coefficients of C-K/S Values for Three Colorants at Different Wavelengths

wavelength	red		yellow		blue	
	function expression	fitting coefficient	function expression	fitting coefficient	function expression	fitting coefficient
400nm	y=0.7136x+0.6580	0.9940	y=1.5910x+0.4882	0.9998	y=0.3706x+0.0492	0.9899
420nm	y=0.6669x+0.2800	0.9899	y=1.6910x+0.4985	0.9998	y=0.2926x+0.0196	0.9995
440nm	y=0.6384x+0.1607	0.9890	y=1.8910x+0.4889	0.9999	y=0.2826x+0.0177	0.9995
460nm	y=0.6207x+0.1797	0.9971	y=1.7010x+0.4227	0.9998	y=0.3626x+0.0160	0.9997
480nm	y=1.0712x+0.2298	0.9990	y=1.2910x+0.3267	0.9997	y=0.3926x+0.0181	0.9997
500nm	y=1.6677x+0.2925	0.9996	y=0.8410x+0.2121	0.9997	y=0.5389x+0.0347	0.9991
520nm	y=2.4677x+0.3590	0.9998	y=0.3910x+0.1126	0.9985	y=0.6789x+0.0393	0.9994
540nm	y=2.6677x+0.3347	0.9998	y=0.1492x+0.0478	0.9999	y=0.8804x+0.0337	0.9992
560nm	y=2.5677x+0.3793	0.9998	y=0.0592x+0.0240	0.9993	y=1.2804x+0.0361	0.9996
580nm	y=1.0735x+0.0464	0.9992	y=0.0242x+0.0098	0.9818	y=1.6753x+0.0568	0.9993
600nm	y=0.2334x+0.0115	0.9958	y=0.0069x+0.0047	0.9995	y=1.8753x+0.0641	0.9995
620nm	y=0.0496x+0.0090	0.9995	y=0.0022x+0.0024	0.9994	y=1.9711x+0.0688	0.9995
640nm	y=0.0097x+0.0068	0.9990	y=0.0010x+0.0020	0.9998	y=1.8953x+0.0598	0.9995
660nm	y=0.0049x+0.0052	0.9997	y=0.0006x+0.0018	0.9995	y=1.2804x+0.0293	0.9996
680nm	y=0.0017x+0.0067	0.9872	y=0.0004x+0.0017	0.9989	y=0.4926x+0.0177	0.9998
700nm	y=0.0007x+0.0064	0.9860	y=0.0003x+0.0015	0.9984	y=0.3426x+0.0046	0.9996

## 4.2. Analysis and Solution of Question 2

### 4.2.1. K-M optical model

For opaque materials, there is a certain relationship between the ratio of absorption coefficient K/scattering coefficient S and reflectivity R. The K-M single constant theory can be directly obtained through the K-M double constant theory, and the specific relationship can be expressed by the following formula (1).

$$\left(\frac{K}{S}\right)_\lambda = \left(\frac{K}{S}\right)_\lambda^0 + \sum_{i=1} c_i \left(\frac{k}{s}\right)_\lambda^i = \frac{(1-R_\infty)^2}{2R_\infty} \tag{1}$$

Among them,  $\left(\frac{K}{S}\right)_\lambda$  is the ratio of the absorption coefficient K and scattering coefficient S of the formula sample at wavelength  $\lambda$ , which is the K/S value;  $\left(\frac{K}{S}\right)_\lambda^0$  is the ratio of the absorption coefficient K and scattering coefficient S of the substrate at wavelength  $\lambda$ ;  $\left(\frac{k}{s}\right)_\lambda^i$  is the ratio of the unit concentration absorption coefficient K and scattering coefficient S of pigment i at wavelength  $\lambda$ ;  $c_i$  is the concentration of colorant i;  $R_\infty$  is the R value of the target sample.

By using the formula (1) of the K-M optical model mentioned above and using Python programming, the K/S values of each target sample at each wavelength can be calculated. Some of the results are shown in Table 2.

Table 2 K/S value data of target samples at different wavelengths (partial)

sample	400nm	420nm	440nm	...	660nm	680nm	700nm
1	0.0616	0.0582	0.0562	...	0.0045	0.0040	0.0034
2	0.0346	0.0276	0.0235	...	0.0202	0.0110	0.0085
3	0.0432	0.0367	0.0331	...	0.0238	0.0129	0.0100
4	0.0483	0.0420	0.0402	...	0.0621	0.0405	0.0327
5	0.0413	0.0347	0.0322	...	0.0530	0.0319	0.0253
6	0.0615	0.0579	0.0556	...	0.0054	0.0043	0.0037
7	0.0732	0.0731	0.0734	...	0.0086	0.0057	0.0047
8	0.0461	0.0421	0.0408	...	0.0202	0.0110	0.0085
9	0.0454	0.0391	0.0369	...	0.0530	0.0320	0.0253
10	0.0674	0.0659	0.0664	...	0.0621	0.0405	0.0328

**4.2.2. Use CIELAB color difference calculation method**

The calculation method for the tristimulus values X, Y, and Z that appear in the calculation of color parameters  $L^*$  (brightness),  $a^*$  (red green) and  $b^*$  (yellow blue) is shown in formula (2). The color parameters  $L^*$ ,  $a^*$ ,  $b^*$  can be obtained from formula (3).

$$\begin{aligned}
 X &= k \int_{400}^{700} S(\lambda)\bar{x}(\lambda)R(\lambda)d(\lambda) \\
 Y &= k \int_{400}^{700} S(\lambda)\bar{y}(\lambda)R(\lambda)d(\lambda) \\
 Z &= k \int_{400}^{700} S(\lambda)\bar{z}(\lambda)R(\lambda)d(\lambda)
 \end{aligned}
 \tag{2}$$

Among them, the product of  $S(\lambda)\bar{x}(\lambda)$ ,  $S(\lambda)\bar{y}(\lambda)$ ,  $S(\lambda)\bar{z}(\lambda)$  are fixed value.  $R(\lambda)$  is the spectral reflectance, with a k value of approximately 0.1.  $d(\lambda)$  is the wavelength interval for measuring the reflectance of the object, and in this question,  $d(\lambda)=20\text{nm}$ .

$$\begin{aligned}
 L^* &= 116\left(\frac{Y}{Y_0}\right)^{\frac{1}{3}} - 16 \\
 a^* &= 500\left[\left(\frac{X}{X_0}\right)^{\frac{1}{3}} - \left(\frac{Y}{Y_0}\right)^{\frac{1}{3}}\right] \\
 b^* &= 200\left[\left(\frac{Y}{Y_0}\right)^{\frac{1}{3}} - \left(\frac{Z}{Z_0}\right)^{\frac{1}{3}}\right]
 \end{aligned}
 \tag{3}$$

Among them,  $X_0$ ,  $Y_0$ ,  $Z_0$  refer to the tristimulus values of ideal white,

$$X_0 = 94.83, Y_0 = 100.00, Z_0 = 107.38$$

By using formula (2) and the weighted table of spectral tristimulus values, using Python programming, the XYZ value of the target sample can be obtained, and then the color

parameters  $L^*$ ,  $a^*$ ,  $b^*$  of the target sample can be obtained from formula (3), as shown in Table 3.

Table 3 Target Sample X, Y, Z Values and Color Parameters  $L^*$ ,  $a^*$ ,  $b^*$  Values

样本	X	Y	Z	$L^*$	$a^*$	$b^*$
1	1151.5275	983.9864	1166.8515	232.5732	77.8093	-14.4156
2	1350.9833	1373.8220	1613.9013	261.8230	14.5862	-14.5639
3	1232.1043	1204.2697	1477.9900	249.8883	29.3790	-20.8777
4	958.4907	1055.4024	1386.6419	238.4470	-15.6920	-30.5188
5	1078.4304	1178.8841	1494.2233	248.0067	-13.5654	-25.8702
6	1131.5603	959.0586	1171.0856	230.4562	80.2532	-18.6009
7	1133.3166	1032.0936	839.5305	236.5599	54.5350	38.4994
8	1312.8427	1342.2994	1414.8333	259.6816	12.3006	2.9341
9	1057.8201	1146.0116	1436.1787	245.5297	-10.0974	-23.8295
10	876.9116	960.6769	1034.9114	230.5947	-13.4302	-0.4579

Use the three attributes of color to comprehensively evaluate the color difference, and use the color difference formula to calculate the visual color difference  $\Delta E^*$  between the two colors, namely color difference. Therefore, the expression for calculating the color difference between the target sample and the formula sample in the CIELAB uniform color space is formula (4).

$$\Delta E^* = \left( \Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2} \right)^{\frac{1}{2}} \tag{4}$$

### 4.2.3. Nonlinear programming model

The ratio of absorption coefficient  $K$ /scattering coefficient  $S$  of each colorant has additive properties. The additive property can be used to mix the three colorants of red, yellow, and blue, and the  $K/S$  value of the obtained formula is expressed as formula (5).

$$\left( \frac{K}{S} \right)_\lambda^0 + \sum_{i=1}^3 c_i \left( \frac{k}{s} \right)_\lambda^i = \left( \frac{K}{S} \right)_\lambda^0 + c_1 \left( \frac{k}{s} \right)_\lambda^1 + c_2 \left( \frac{k}{s} \right)_\lambda^2 + c_3 \left( \frac{k}{s} \right)_\lambda^3 \tag{5}$$

The first step is to establish the objective function. The minimization function of the color difference between the target sample and the formula sample is used to calculate the Euclidean distance between the  $R$  value of the target sample and the  $R$  value of the formula sample. In this paper, the  $R$  value is replaced by the  $K/S$  value to establish the objective function, which is formula (6).

$$\min Z = \sqrt{\left[ \left( \frac{K}{S} \right)_\lambda - \left( \left( \frac{K}{S} \right)_\lambda^0 + c_1 \left( \frac{k}{s} \right)_\lambda^1 + c_2 \left( \frac{k}{s} \right)_\lambda^2 + c_3 \left( \frac{k}{s} \right)_\lambda^3 \right) \right]^2} \tag{6}$$

Step 2, set constraints. Color additive concentration constraint conditions: Two constraint conditions are that the concentration of each color additive must be greater than or equal to 0 and the sum of the concentrations of each part of the color scheme is 1; Color difference constraint condition: A constraint condition where the color difference between the target sample and the formula sample is less than 1.

$$s.t. \begin{cases} c_i \geq 0, i = 1, 2, 3 \\ \sum_{i=1}^3 c_i = 1 \\ \Delta E^* = (\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2})^{\frac{1}{2}} < 1 \end{cases} \quad (7)$$

The third step is to integrate the objective function and constraint conditions to form a nonlinear programming model, optimize and solve to obtain the concentration formula. Use the simpson function of scipy. integrate to calculate the color parameters  $L^*$ ,  $a^*$ ,  $b^*$  of each optimal formula sample, and substitute them into formula (4) to obtain the color difference between the objective sample and the formula sample, as shown in Table 4.

Table 4 The optimal formula and color difference of the target sample

sample	c1	c2	c3	chromatism
1	0.7945	0.0919	0.1136	0.6041
2	0.1348	0.3295	0.5357	0.8802
3	0.3107	0.2271	0.4622	0.7927
4	0.0000	0.0463	0.9537	0.6605
5	0.0000	0.1127	0.8873	0.7465
6	0.8558	0.0304	0.1138	0.5935
7	0.2481	0.6627	0.0892	0.4686
8	0.0761	0.4996	0.4242	0.8045
9	0.0000	0.1317	0.8683	0.7189
10	0.0000	0.3366	0.6634	0.5235

### 4.3. Analysis and Solution of Problem Three

Firstly, establish a nonlinear programming model.

The first step is to establish the objective function. Considering both color difference and cost comprehensively, namely the minimization function of color difference and cost sum between the target sample and the formula sample. Based on the unit weight price of color masterbatch, the objective function is established by adding cost control objectives to the objective function formula (6) in question 2, which is formula (7).

$$\begin{aligned} \min Z &= \sqrt{\left[ \left( \frac{K}{S} \right)_\lambda - \left( \left( \frac{K}{S} \right)_\lambda^0 + \sum_{i=1}^3 c_i \left( \frac{k}{s} \right)_\lambda^i \right) \right]^2} + G \left( \sum_{i=1}^3 p_i c_i \right) \\ &= \sqrt{\left[ \left( \frac{K}{S} \right)_\lambda - \left( \left( \frac{K}{S} \right)_\lambda^0 + c_1 \left( \frac{k}{s} \right)_\lambda^1 + c_2 \left( \frac{k}{s} \right)_\lambda^2 + c_3 \left( \frac{k}{s} \right)_\lambda^3 \right) \right]^2} + G(p_1 c_1 + p_2 c_2 + p_3 c_3) \end{aligned} \quad (8)$$

Among them,  $p_i$  is the unit weight price of the color master batch,  $p_1 = 60$ ,  $p_2 = 65$ ,  $p_3 = 63$ , and G is the weight of the base material.

$$s.t. \begin{cases} c_i \geq 0, i = 1, 2, 3 \\ \sum_{i=1}^3 c_i = 1 \\ \Delta E^* = \left( \Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2} \right)^{\frac{1}{2}} < 1 \\ G = 2 \end{cases}$$

The third step is to integrate the objective function and constraint conditions to form a nonlinear programming model, optimize and solve to obtain the concentration formula. Use the scipy-integrate simpson function to calculate the color parameters  $L^*$ ,  $a^*$ ,  $b^*$  of each optimal formula sample, and substitute them into formula (4) to obtain the color difference between the objective sample and the formula sample, as shown in Table 5.

Table 5 The optimal formula and color difference of the target sample

sample	c1	c2	c3	chromatism
1	0.8771	0.0185	0.1044	0.6080
2	0.1869	0.2812	0.5319	0.8828
3	0.3774	0.1725	0.4501	0.7958
4	0.0000	0.0227	0.9773	0.6609
5	0.0000	0.0833	0.9167	0.7471
6	0.8958	0.0000	0.1042	0.5944
7	0.2761	0.6358	0.0880	0.4700
8	0.1137	0.4607	0.4256	0.8065
9	0.0000	0.1024	0.8976	0.7195
10	0.0000	0.3186	0.6814	0.5239

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