

# Study on hydropower distribution system and its optimization analysis considering the current situation of water resources supply

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## Abstract

In today's increasingly scarce water resources, it is particularly important to plan and allocate the water stored in reservoirs. This paper addressed regarding the allocation of hydroelectric power at two dams in the western United States. we create an optimal model to allocate water resources and an additional water use function. by means of the equation we can obtain the amount of water withdrawn from each of the two reservoirs. we build a model to allocate water to the highest sum of profit from agriculture and water generation after the allocation of water, and inverse solve the reservoir's change height by the optimization equation, the amount of water allocated.

## Keywords

Utilities distribution; multi-objective planning; economic effect.

## 1. Introduction

### 1.1. Prospects for intelligent robots

Water is an indispensable material for people's survival and social development. With the change of geography and climate, the real fresh water resources available globally are becoming less and less and are accompanied by unevenness in time and space, and the increase of population along with the intervention of human activities also makes the water consumption increase rapidly [1]. The rational planning and distribution of water resources has become a common problem that we now need to face.

The construction of dams on rivers and streams has become a significant project in order to better deal with the problem of human water supply, and the dams have a wide range of functions, such as maintaining water for the population and generating electricity from the falling water [2-3]. Today, in the Colorado River basin in the United States, there is a prolonged drought, and the U.S. government has established Glen Canyon Dam (Lake Powell) and Hoover Dam (Lake Mead) near the river basin to ensure that the residents around the river have a convenient life, sufficient water for industry and agriculture, and the ability to produce electricity for the residents [4].

After the climate has affected the reservoir's water supply, the amount of water has been reduced to the point where the water supply cannot be met. Our team needs to build a model to analyze the specific living conditions of the people living in the vicinity of the Colorado River and the development of agriculture and industry in the area, and to make a rational and defensive plan for the allocation of water from Glen and Hoover dams for present and future supply [5].

## 2. Establishment of model

### 2.1. Optimal water withdrawal allocation model

#### 2.1.1 Finding and processing of correlation coefficients

In order to have a better understanding of the specific environment for water allocation, our team consulted various official websites and more authoritative literature, including the USGS, among others. According to Table 1.

Table 1: Related data sheets

Noun Name	Data
$W_t$ total precipitation ( $m^3$ ).	$5.336 \times 10^9$ ( $m^3$ /month)
Total water demand of W residents ( $m^3$ ).	$1.311 \times 10^{10}$ ( $m^3$ /month)
The $V_d$ Glen Canyon Dam flows into volume ( $m^3$ ) per month	$8.2308 \times 10^8$ ( $m^3$ /month)
Lake $V_r$ Mead has a monthly outflow volume ( $m^3$ ).	$1.99728 \times 10^8$ ( $m^3$ /month)
The volume of $\Delta V$ flowing into the two reservoirs	$6.23352 \times 10^8$ ( $m^3$ /month)
$S_H$ Lake Mead area ( $km^2$ ).	640
$S_G$ Lake Powell area ( $km^2$ ).	658.12
Income from $M_A$ agricultural irrigation (yuan/ $m^3$ ).	2 ( $m^3$ /month)
Lake Mead's highest water levels are $A_H$ and stagnant water levels $B_H$ ( $m$ ).	374.6, 289.6
The highest water level of Lake Powell is $A_G$ ( $m$ ) and the stagnant water level $B_G$ ( $m$ ).	228.6, 66.3

It is important to emphasize that for agricultural irrigation returns we use the irrigation of wheat as a guide data, i.e. to build a connection between the amount of wheat that needs to be watered within a unit and the normal return of that unit area of wheat.

1 mu of wheat ~ need to water 80m<sup>3</sup> of water ~ can yield 160 yuan

#### 2.1.2 Relationship management between population and water

According to relevant reports: The water used by the population (agricultural, industrial and domestic) in these USA western states comes mainly from natural rainfall, groundwater uptake and withdrawal in dam reservoirs, but because the source of groundwater eventually collects in rivers and streams, and the flow of streams and rivers collects in dam reservoirs (shown in Figure 1 below), and because of factors such as pressure and stress, we can consider groundwater pumping as water withdrawn in reservoirs. The extracted water. When the groundwater is extracted the reservoir water will naturally fall, then the fall of the reservoir water level is the sum of the reservoir extraction and the groundwater.



Figure 1: Groundwater sources

And because the accumulation of precipitation and reservoir water are obtained by multiplying the area and water depth, the total water consumption of people in the five states is the amount

of rainfall and the reduction of water in the two reservoirs, and the water consumption relationship can be established:

$$W = W_S + S_G h_G + S_H h_H \tag{1}$$

Due to climate reasons the amount of precipitation in the sky has been decreasing in recent years, and the amount of water saved in the dam reservoir is also decreasing (as shown in Figure 2,3), but people's demand for water is not going to change, we have to consider when the precipitation and other additional water in addition to the reservoir is 0 in the case of our reservoir water can still maintain the residents water for a long time.

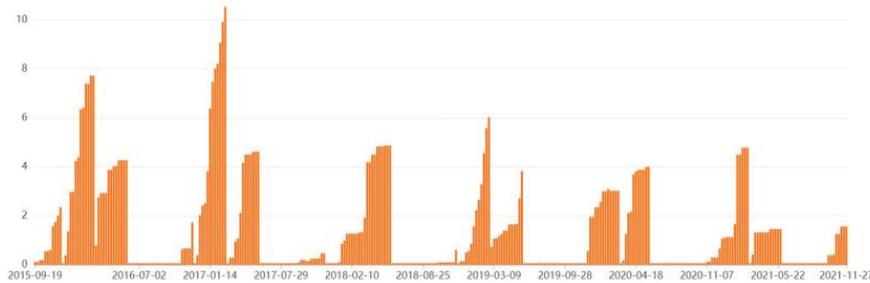


Figure 2: Diagram of annual precipitation accumulation in California for the last six years

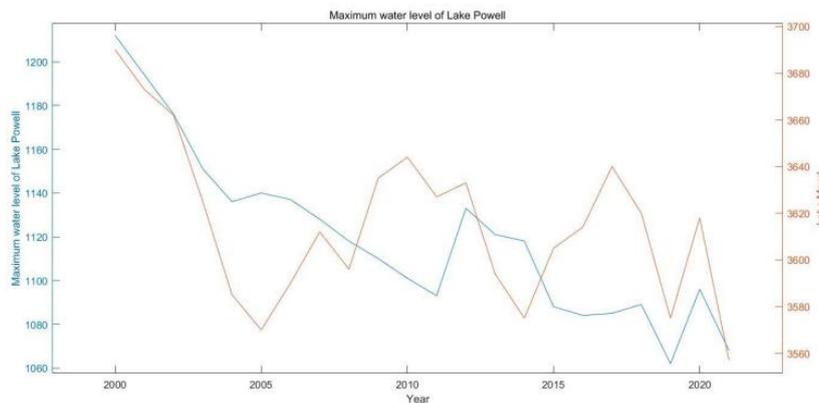


Figure 3: Water level changes in Lake Mead and Lake Powell

According to authoritative information, in the western states of the United States, 80% of the residents' water consumption is used for crop irrigation, and only 20% of the water is used for the residents' daily life and industrial water, at this time we use the water in the reservoir for as long as possible, in which the people of the United States only consider life and a small portion of industrial water, in the case of fixed water for residents, that is, the time and water stock water to form a positive ratio.

$$T_C = \frac{S_G(P - A_G) + S_H(M - A_H) - V_r}{W * 20\%} \tag{2}$$

The time  $T_C$  is equal to the ratio of the total water stock, excluding the water to be discharged to the Gulf of California in the Mexico basin, to the water used by the population for domestic purposes. In order to achieve a sustainable development, the water level of the dam reservoir should be kept at the minimum level, and at the same time, the water in the reservoir should be restored as quickly as possible, so that the local effect can be optimized.

Because in the acquisition of water resources reservoirs in the process of rapid recovery can quickly replenish the source of water is nature's precipitation storage and upstream river interception, and in our assumption of precipitation and upstream water supply are the average of recent years, so over time to meet the fixed water demand, the amount of additional water must be replenished with the time shift, the formula can be expressed as follows:

$$Q = t(W_S + V_d) \tag{3}$$

### 2.1.3 Establishment of the optimal water withdrawal allocation model

The water in Lake Mead continues to flow down to the field of Mexico, the need to consider the rights of Mexico, when the water level of the reservoir of Lake Mead will cause the Mexican river will be lowered, in order to maximize the protection of the United States and Mexico both have more adequate water at the same time also need to protect the safety of the reservoir, when the water level of the reservoir and the river level is too much difference will exist risk factors, then in a certain period of time to reduce The loss of water is minimal. Therefore, only the objective function is required to minimize the loss of water in Mexico to obtain the global optimal water withdrawal height.

$$\min f_1 = h_H l_H V_r \tag{4}$$

The reservoirs have a dead level AG and AH (this level does not support continued power generation) and an overflow level BG and BH (the highest level) through the equation we can get the amount of water flowing to Mexico and the amount of water pumped by the two reservoirs to ensure that the needs of the US population are met. Therefore, the constraint equation is:

$$\text{st.} \begin{cases} N = S_G h_G + S_H h_H - V_r \\ A_G < P - h_G < B_G \\ A_H < M - h_H < B_H \end{cases} \tag{5}$$

## 2.2. Improvement of optimal allocation water withdrawal

### 2.2.1 Develop an improved model for optimal water withdrawal allocation

Our aim is to maximize the sum of electricity money and water money in agriculture, all with an objective function of

$$\max Z = E(T) + E(F) \tag{6}$$

First of all, the water level should not exceed the minimum water level.

$$A_G < P - h_G < B_G \tag{7}$$

$$A_H < M - h_H < B_H \tag{8}$$

The reservoir needs to supply water out due to the shortage of water needed to supply water to the population in the early

$$h_G > 0, h_H > 0 \tag{9}$$

When the volume of water used in the countryside is less than the total volume of water needed in the countryside ( $W_A < V_F$ )

$$E(F) = \frac{W_A}{V_F} \cdot M_A \tag{10}$$

When the volume of water used in the countryside is greater than the total volume of water needed in the countryside ( $W_A > V_F$ )

$$E(F) = M_A \tag{11}$$

The power generation effect is only related to the height of the water surface, by the difference between the present water surface and the lowest water level divided by the maximum amount of water is the effective working rate of the engine, and through data collection, the total working efficiency of different reservoirs is obtained, so the total governor power generation effect can be obtained

$$E(T) = \frac{(M-h_H)-A_H}{B_H-A_H} \cdot W_{EH} + \frac{(P-h_G)-A_G}{B_G-A_G} \cdot W_{EG} \tag{12}$$

Based on this, an improved model for optimal water withdrawal allocation was developed.

The model only requires the satellite to regularly check the water level, input  $M$  and  $P$ , you can get the height of the water surface of the lake drop  $h_G$ ,  $h_H$ , and then know how much water is taken from the reservoir, how much effect of hydroelectricity respectively maximum.

The final overall model can be:

$$\max Z = E(T) + E(F) \quad (13)$$

$$E(T) = \frac{(M-h_H)-A_H}{B_H-A_H} \cdot W_{EH} + \frac{(P-h_G)-A_G}{B_G-A_G} \cdot W_{EG} \quad (14)$$

$$E(F) = M_A (W_A > V_F) \quad (15)$$

$$E(F) = \frac{W_A}{V_F} \cdot M_A (W_A < V_F) \quad (16)$$

$$h_G > 0, h_H > 0 \quad (17)$$

$$A_G < P - h_G < B_G \quad (18)$$

$$A_H < M - h_H < B_H \quad (19)$$

$$A_H < M - h_H < B_H \quad (20)$$

$$S_G h_G + S_H h_H + V_d = V_r + N + V_F \quad (21)$$

### 3. Solution of model

#### 3.1. Model for guiding dam operation

In order to obtain the amount of water pumped from the two different dams, respectively, by the flow of Lake Mead to Mexico field river flow restrictions, if we want to get the target equation to get the minimum value, is the target equation of the height of the water in the Glen Dam when falling to get the minimum value, it should be noted here is that the topic was initially given the water level of Lake Powell and Lake Mead line After the data search and the description of the data in the previous article, we can learn that Lake Powell is an artificial lake, so the capacity of Lake Mead is much larger than Lake Powell, while the minimum water level of Lake Mead is also higher than the maximum water level of Lake Powell, then there is  $M > P$  constant holds.

By solving the equation consisting of the objective function and the constraints to obtain  $h_G$  (in meters)

$$h_G > 261.02 + \frac{P-0.977M}{2} \quad (22)$$

The minimum value of  $h_G$  is transformed as the initial values provided by the two reservoirs transform, and because the sum of the water taken by the reservoirs is a constant value, it is obtained that when

$$h_G = 261.02 + 0.5(P - 0.977M) \quad (23)$$

You can get:

$$h_H = 0.502M - 0.514P - 50.6 \quad (24)$$

Only the reservoir can provide water to the residents and there is no any other source of water. The time  $T_C$  (in months) that can be maintained is obtained by calculating the ratio of the reservoir capacity to the demand:

$$T_C = 2.51P + 0.244M - 87.33 \quad (25)$$

The time to be able to maintain water for residential use is obtained as the initial water level line of the two reservoirs changes.

The amount of water that must be additionally replenished over time to meet a fixed water demand changes with time, and the formula can be expressed as  $Q$  (cubic meters):

$$Q = t(W_s + V_d) = t * (5.336 * 10^9 + 273600) \quad (26)$$

### 3.2. The best solution for the conflict between water for general use and water for power generation

Through the analysis and modeling of the conflict between water for general use and water for power generation, and the impact of three scenarios. We need to determine and solve for the initial water level  $M$  and  $P$ , and the water allocation between agricultural irrigation and water for power generation in order to maximize the economic benefits of water resources in addition to residential water use. Because the benefit of electricity generated by water is much greater than that of agricultural irrigation without considering the cost of the same volume of water used for power generation, we need to give priority to the greater benefit of  $E(T)$  in solving the equation:  $M_H - h_H$  and  $P_G - h_H$  are as large as possible, and then the following relationship is obtained by solving the equation.

$$\begin{cases} P - h_G = 215 \\ M - h_H = 367.8 \end{cases} \quad (27)$$

Because after satisfying more benefits of water for electricity generation, in order to satisfy the target equation of maximizing the sum of benefits of electricity generation and benefits of agricultural irrigation,

$$\textit{it is obtained that} \begin{cases} h_H = 13.6, M = 228.6 \\ h_G = 6.8, P = 374.6 \end{cases} \quad (28)$$

When there is a conflict between general water use and water for power generation, we need to consider it comprehensively. In the above solution, we know that when water resources are not enough, we need to increase the water for power generation as much as possible while satisfying people's basic water for living.

The Department of Natural Resources also asked our team to come up with a solution to the shortage of water for people's living and power generation, since the decline of local water resources is an important phenomenon that accompanies natural climate change. However, the western U.S. is near the Pacific Ocean, and methods and technologies for freshwater use of seawater are constantly being updated, and such methods are supplementing the population's water supply to some extent. However, it is not yet possible to solve the problem of severe water shortage in the western part of the United States, which is still in a state of water shortage as can be observed in the figure 4 below. At the level of water-to-power, the U.S. has adopted other modes of power generation to obtain more electricity, for example, in the California Energy Commission's data, California accounted for 34% of renewable energy in 2018. In addition to increasing the use of other energy sources to generate electricity, there is a need to reduce the amount of water people use on a daily basis to some extent in order to conserve water and contribute to global water resources.



Figure 4: Map of water sources in selected regions of the Americas

## 4. Conclusion

### 4.1. Conclusion of the model for guiding dam operation

By solving the equation consisting of the objective function and the constraints we obtain:

$$h_G = 261.02 + 0.5(P - 0.977M) \tag{29}$$

$$h_H = 0.502M - 0.514P - 50.6 \tag{30}$$

The time (in months) that can be maintained is obtained by calculating the ratio of reservoir capacity to demand.

$$T_C = 2.51P + 0.244M - 87.33 \tag{31}$$

Additional supplemental water volume functions

$$Q = t(W_S + V_d) = t * (5.336 * 10^9 + 273600) \tag{32}$$

### 4.2. Conclusion of the best solution for the conflict between water for general use and water for power generation and the impact of three scenarios

When the reservoir water supply is less than the people's survival needs water  $N$ , the water should all meet the human life needs.

When the reservoir water supply is greater than the people's survival demand water  $N$

With the model and the establishment of our assumed initial water level  $M$  with  $P$  the following relationship was obtained.

$$\begin{cases} P - h_G = 215 \\ M - h_H = 367.8 \end{cases} \tag{33}$$

Because after satisfying more benefits of water for electricity generation, in order to satisfy the target equation of maximizing the sum of benefits of electricity generation and benefits of agricultural irrigation, it is obtained that

$$\begin{cases} h_H = 13.6, M = 228.6 \\ h_G = 6.8, P = 374.6 \end{cases} \tag{34}$$

When there is a conflict between general water use and water for power generation, we need to consider it comprehensively. In the above solution, we know that when water resources are not enough, we need to increase the water for power generation as much as possible while satisfying people's basic water for living.

The overall model is:

Objective function:

$$\max Z = E(T) + E(F) \tag{35}$$

When the constraints are

$$E(T) = \frac{(M-h_H)-A_H}{B_H-A_H} \cdot W_{EH} + \frac{(P-h_G)-A_G}{B_G-A_G} \cdot W_{EG} \quad (36)$$

$$E(F) = M_A (W_A > V_F) \quad (37)$$

$$E(F) = \frac{W_A}{V_F} \cdot M_A (W_A < V_F) \quad (38)$$

$$h_G > 0, h_H > 0 \quad (39)$$

$$A_G < P - h_G < B_G \quad (40)$$

$$A_H < M - h_H < B_H \quad (41)$$

$$A_H < M - h_H < B_H \quad (42)$$

$$S_G h_G + S_H h_H + V_d = V_r + N + V_F \quad (43)$$

By measuring the elevation by satellite and inputting the depths of  $P$  and  $M$  reservoirs, the optimal solution that allows the combined effect to maximize the two water level changes is obtained.

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