

Research on Fuzzy Algorithm and Its Decision Expansion

Ruiqi Liu

Jinan Foreign Language School, Jinan City, China

Abstract

Because of the uncertainty of objective things and the fuzziness of decision makers' thinking, decision information is sometimes given in the form of triangular fuzzy numbers. A simple calculation method of fuzzy number addition, subtraction, multiplication and Division is given. Aiming at the stochastic multi-criteria decision-making problem with intuitionistic fuzzy numbers and known criteria weights, a decision-making analysis method based on improved prospect theory is proposed. Combined with the improved prospect decision matrix and criteria weights, the comprehensive utility value of each scheme is calculated, and the order of the schemes is determined. Finally, an example is given to verify the effectiveness and rationality of the proposed intuitionistic fuzzy stochastic multi-criteria decision-making method, and the basic concept, basic operation and operation law of fuzzy sets are introduced. In the application of fuzzy sets, fuzzy preference relation is introduced, and the concept of uniform fuzzy judgment matrix is extended. Finally, based on the concept of the generalized uniform fuzzy judgment matrix proposed by us, an optimization model is established to be used in decision-making problems. The effectiveness and feasibility of that method are illustrate by an example.

Keywords

Fuzzy number, Algorithm, Fuzzy set, Decision-making.

1. Introduction

Mathematics is to make mathematics contain a hazy phenomenon area, not to say that mathematics becomes indistinct and indistinct. This kind of fuzzy mathematics is used to quantitatively study and deal with "fuzzy" problems. There are many objective things. This kind of phenomenon, it is difficult to clearly define the boundaries. The application of fuzzy number theory can solve the problems existing in the traditional probability theory. For those basic events with low probability of failure and no accurate value of probability of failure, it is considered that the probability of occurrence of these basic events is fuzzy number, that is, fuzzy probability is used to describe the failure behavior of the system and its components, which makes the risk evaluation results more reasonable and scientific [1]. Judging from the current Pythagorean fuzzy number ranking method, it seems that it is effective in solving the multi-attribute decision-making problem in Pythagorean fuzzy environment, but there are some shortcomings. Intuitionistic weighted average operator and mixed aggregation operator measure risk, and the determination of normal weight is completely subjective [2]. The parameter fuzzification ability of option model based on trapezoidal fuzzy number is too low. In practical application, it is often inconvenient to study the intuitionistic fuzzy set as a whole, and we often separate its elements for research, so the concept of intuitionistic fuzzy number comes into being [3]. The rules of fuzzy number addition, subtraction, multiplication and division have existed for a long time, but these rules are rather general, and it is very troublesome to calculate in practice. This paper simplifies them and mainly discusses the application of the simplified rules. Because of the uncertainty of objective things and the fuzziness of decision makers' thinking, decision information is sometimes given in the form of

triangular fuzzy numbers [4]. A decision-making method based on fuzzy proportional value and fuzzy deviation degree is proposed for the multi-attribute decision-making problem, in which the preference of decision makers is known, the attribute values are given in the form of triangular fuzzy numbers, and the attribute weight information is not completely known. In the research results of triangular fuzzy numbers, the denominator of group utility value or individual regret value will not be negative, so it does not violate the division rule of triangular fuzzy numbers. A new representation of intuitionistic fuzzy numbers is given, and the utility function of intuitionistic fuzzy numbers is introduced through the new representation [5]. In this way, the intuitionistic fuzzy number has a quantitative utility measure, which the previous dictionary method does not have. Of course, with the utility function, the comparison of intuitive fuzzy numbers only needs to compare the utility of intuitive fuzzy numbers [6].

2. Fuzzy set and its operation

2.1. The concept of fuzzy set

Since the establishment of set theory, set has become one of the most basic and commonly used methods and concepts in modern mathematics. Set theory has been the foundation of modern mathematics, and has quickly penetrated into other branches of mathematics and related disciplines [7]. A set can be defined as a collection of objects or things that have certain properties, are definite and can be distinguished, and can be regarded as a whole. The concept described is deterministic, that is, the elements in a set are deterministic. An object either belongs to this set or does not belong to this set. The two must be one or the other, and there is no other situation. Fuzzy set is an extension of the concept of ordinary set. When the value range of membership function of fuzzy set is in a certain interval, it will degenerate into ordinary set [8]. The correlation coefficient of interval intuitionistic fuzzy numbers is introduced into uncertain multi-criteria decision making, and there is an equivalent relationship between the correlation coefficient and similarity. According to the maximization of the correlation coefficient between the decision object and the ideal scheme, a nonlinear programming model is constructed.

The normal convex fuzzy set on the real number set R is called fuzzy number. In this paper, the convex fuzzy set on the real number set R is called fuzzy number, that is, the "normality" is removed. The fuzzy numbers on the closed interval $[e, f]$ of the universe of discourse are recorded as $A[e, f]$. The fuzzy number is called isosceles fuzzy number, and $I^\lambda[a, b]$ is shown in formula (1).

$$A[a, b](x) = \frac{2^\lambda}{b-a} (x-a), x \in [a, \frac{a+b}{2}] \quad (1)$$

All isosceles fuzzy numbers are denoted as δ , and the isosceles fuzzy numbers are shown in Figure 1.

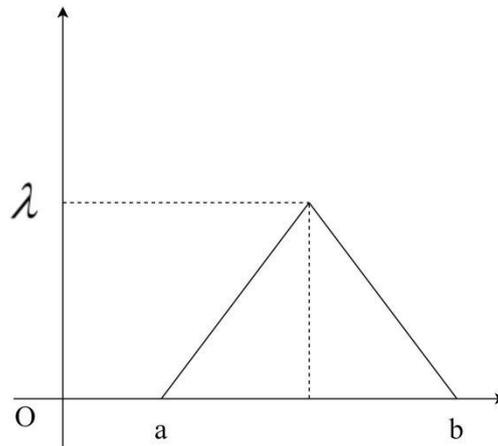


Figure 1 Isosceles fuzzy number

2.2. Algorithm of fuzzy set

In ordinary sets, there are equality and inclusion relations between sets, and operations include remainder, union and intersection. Similarly, the relation or operation on the corresponding fuzzy set can be defined by the membership function of the fuzzy set [9]. Fuzzy sets have similar properties to ordinary set operations: In ordinary sets, there are equality and inclusion relations between sets, and operations include remainder, union and intersection. Similarly, the relation or operation on the corresponding fuzzy set can be defined by the membership function of the fuzzy set [9]. Fuzzy sets have similar properties to ordinary set operations:

- (1) Idempotent law: $\tilde{A} \cup \tilde{A} = \tilde{A}, \tilde{A} \cap \tilde{A} = \tilde{A}$
- (2) Exchange Law: $\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A}, \tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$
- (3) Law of association: $(\tilde{A} \cup \tilde{B}) \cup \tilde{C} = \tilde{A} \cup (\tilde{B} \cup \tilde{C}), (\tilde{A} \cap \tilde{B}) \cap \tilde{C} = \tilde{A} \cap (\tilde{B} \cap \tilde{C})$
- (4) Distribution Law: $\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C}), \tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$
- (5) Absorption law: $(\tilde{A} \cup \tilde{B}) \cap \tilde{A} = \tilde{A}, (\tilde{A} \cap \tilde{B}) \cup \tilde{A} = \tilde{A}$
- (6) Restoration Law: $(\tilde{A}^c)^c = \tilde{A}$
- (7) Law of duality: $(\tilde{A} \cup \tilde{B})^c = \tilde{A}^c \cap \tilde{B}^c, (\tilde{A} \cap \tilde{B})^c = \tilde{A}^c \cup \tilde{B}^c$

Fuzzy mathematics has been widely used in many fields and achieved fruitful results. For example, fuzzy relation, fuzzy clustering, fuzzy programming, fuzzy decision and fuzzy control.

3. Decision extension of fuzzy numbers

3.1. Application of Fuzzy Number in Fault Tree Analysis

Advantages of fault tree: First, the fault tree analysis method can analyze the units, subsystems and various parts in the system, and at the same time, it can analyze various failures of the system, as well as the influence of parts, units and subsystems on the system. This method has great flexibility in use. Secondly, the fault tree analysis method is a logical causality diagram. By establishing a fault tree, we can intuitively and clearly see the relationship between each unit and what is the mutual influence. Moreover, it is easy to find the relationship between internal factors through the direct diagram. By analyzing the shortcomings of the system, we can optimize and improve it, so that the system becomes more and more reliable. Thirdly, the fault tree analysis method requires that the logic of each bottom element must be correctly known, and then the occurrence probability of the top event can be obtained by substituting the data

into the relevant formula, so as to prepare for the reliability of the following system. Fuzzy numbers are used to give the probability value of major dangerous equipment failure events, and fuzzy numbers are used to represent the probability of basic events, so as to reduce the difficulty of obtaining the accurate probability value of events [10]. Using fuzzy probability method and combining with the actual experience and judgment of engineers and technicians to construct the membership function of fuzzy numbers, it can have great adaptability in the process of describing and evaluating major dangerous equipment. In this paper, taking an atmospheric furnace in a refinery as an example, the fuzzy fault tree analysis method is used to evaluate its risk. Taking the fire and explosion of atmospheric furnace as the top event, the fault tree is established as shown in Figure 2, with A as the top event, B, C, D, E represents the intermediate event, and x_i represents the basic event. According to the statistics of the actual operation status of the field equipment and the expert experience, the basic event x_i , $i = 1, 2, \dots, 13$, The occurrence probability is triangular fuzzy number $[a, b]$ on the closed interval $[a, b]$, and $i = 1, 2, \dots, 13$ is listed in Table 1: the basic events are independent of each other, and the truth function of the intermediate events can be known from Figure 2.

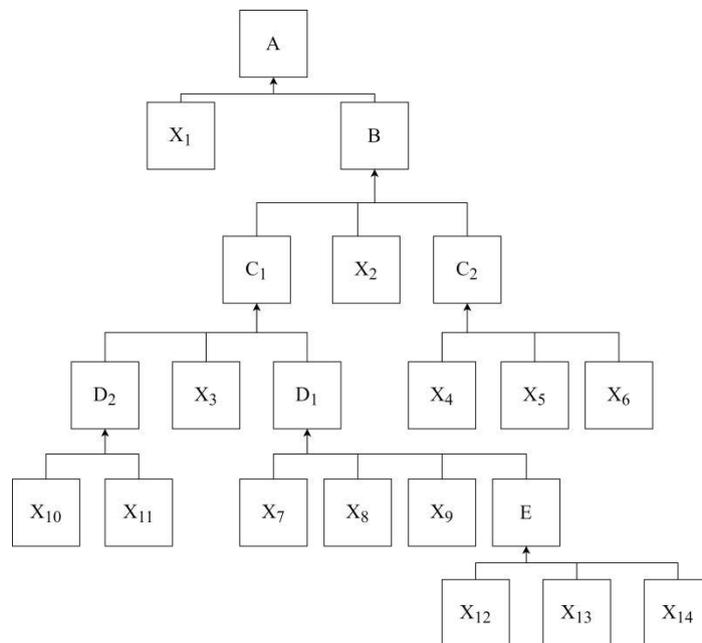


Figure 2 Fault tree

The fuzzy probability theory is introduced into the fault tree analysis of atmospheric furnace, and the fuzzy number is used to describe the probability of event occurrence, which reduces the difficulty of obtaining the accurate value of event occurrence probability. By using the existing statistical data and the fuzzy number of equipment fault occurrence probability, the difference between fault occurrence probability and risk degree can be calculated.

3.2. Fuzzy judgment matrix

One of the main applications of fuzzy sets in decision-making is fuzzy preference relation or fuzzy judgment matrix [11]. For the convenience of the following description, we define a set of natural number set representation schemes. Because the decision-makers usually deviate when comparing various schemes. If the comparison between different schemes by decision makers is completely consistent, or there is no contradiction, then the fuzzy judgment matrix given by decision makers is consistent. General consistency fuzzy judgment matrix is the natural generalization of additive consistency fuzzy judgment matrix and multiplicative consistency fuzzy judgment matrix. If the decision maker has given a fuzzy preference relation or fuzzy

judgment matrix, then the next thing to do is how to make a decision according to the matrix, that is, get a weight vector by a certain method or model. Decision-making methods based on additive consistency and multiplicative consistency have been studied and discussed by many scholars in many literatures. This paper will build a model based on our concept of general consistency. We can regard preference deviation matrix as the decision-making habit of decision makers. The closer it is, the clearer and more positive the attitude of decision makers is. The closer it is, the more ambiguous and eclectic the attitude of the decision maker is [12]. However, the decision-making habits of a decision-maker should not change too much, and the decision-making habits should have certain consistency, that is, the differences of elements in the matrix should be as small as possible.

3.3. Pythagorean fuzzy number

In fuzzy sets, each element in the universe of discourse is given a degree of membership, that is, the degree to which the element belongs to a certain fuzzy set [13]. In the intuitionistic fuzzy set, each element will be given two values: membership and non-membership. Among them, the degree of membership still indicates that an element belongs to an intuitionistic fuzzy set, while the degree of non-membership indicates that it does not belong to an intuitionistic fuzzy set.

With the deepening of research, we find that there are some defects in intuitionistic fuzzy sets. In the process of decision-making, it is assumed that the decision maker's degree of support for something is $\frac{\sqrt{3}}{2}$, but the degree of opposition is $\frac{1}{2}$, and the sum of support and opposition exceeds 1, which contradicts the constraint condition of intuitionistic fuzzy sets: the sum of membership and non-membership is less than 1. Therefore, on the basis of studying the complementary operation of fuzzy sets and intuitionistic fuzzy sets, Pythagorean fuzzy complementary operation was defined in 2014, and it was proposed that the sum of allowable membership and non-membership exceeds 1. And the concept of Pythagorean fuzzy sets whose square sum is less than 1, Pythagorean fuzzy sets have stronger ability to describe fuzzy phenomena than intuitive fuzzy sets, which is a generalization of intuitive fuzzy sets [14]. The strict ordering of Pythagorean fuzzy numbers is put forward, the similarity measure of Pythagorean fuzzy numbers is defined, and a simple and effective Pythagorean multi-attribute decision-making method is put forward. The subtraction and division operations are defined, the precision function is put forward, and the ranking method of Pythagorean fuzzy numbers is put forward. The interval-valued Pythagorean fuzzy set and its decision-making application are studied.

4. Conclusion

Based on fuzzy sets, this paper puts forward intuitionistic fuzzy sets with probability and intuitionistic fuzzy numbers with probability, and defines the algorithm of intuitionistic fuzzy numbers with probabilistic hesitation. The distance measure between two intuitionistic fuzzy numbers with probabilistic hesitation is given, and its related properties are analyzed. Furthermore, a multi-attribute decision-making method based on intuitionistic fuzzy sets is proposed. Aiming at the multi-criteria decision-making problem of interval intuitionistic fuzzy numbers with completely unknown information, the correlation coefficient of interval intuitionistic fuzzy numbers is introduced to overcome the information confusion caused by distance measurement, and the information loss can be reduced to the greatest extent by considering the factors of hesitation. Combining the prospect decision matrix and criterion weight model, the decision method to determine the optimal scheme is given. In the future research, the stochastic multi-criteria decision-making problem with interval hesitant fuzzy

numbers can be further considered. A new algorithm of Pythagoras fuzzy number and its corresponding properties are obtained. The research in this paper makes the Bayesian fuzzy operation more realistic and lays a foundation for the further study of Bayesian fuzzy numbers.

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