

## A new carbon sequestration model based on grey relational analysis optimization model

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### Abstract

In this paper, the grey correlation degree fitting was carried out for the variety of forest characteristic index number, forest number, age ratio, and the index of forest offspring and the area, precipitation, and air temperature representing the amount of solid state. The relationship between forest area and time was obtained through the construction of function relationship, and the carbon sequestration model ( Formula 2 ) was further obtained. The model was discrete, and it needed to be solved through discretization. Finally, the best forest management mode is obtained by optimization. Meanwhile, we set five indicators of forest volume, water storage, carbon sequestration, forest oxygen release and sand control benefits for multi-objective planning and fuzzy multi-objective evaluation. On this basis, a decision model is constructed. And by setting the harvest area parameter to zero, to determine the conditions can not harvest the forest. Establish critical points for increasing and decreasing forest benefits. Transition points applicable to the management plan. Transition points between management plans can be determined by considering tree species, age composition and climatic zones between different forests. In addition, when the constructed model is applied to the determined forest, it is calculated that this third-grade product will reduce about 32.9882 Tg CO<sub>2</sub> within 100 years. This forest needs to adopt a plan of planting and harvesting cycle of one year, planting area of 20,000 mu and harvesting area of 20,000 mu. Since the forest is about to reach the environmental capacity, a shorter planting and harvesting cycle is adopted. At the same time, considering the ten-year extension of the harvest time in the scheme and the comprehensive consideration of the sensitivity of forest managers and other people, this paper obtains a scheme that shortens the planting cycle to 0.65 times, maintains the original harvest plan in the first two decades, increases the harvest cycle by 0.2 times per year from the beginning of the 21st year, and increases the harvest by 0.15 times, which can minimize the benefit within 33 years.

### Keywords

Discrete function of fuzzy multi-objective evaluation, multi-objective programming, grey relational analysis optimization model.

## 1. Introduction

### 1.1. Background

Nowadays, excessive emissions of greenhouse gases such as carbon dioxide lead to global warming. In order to improve the climate conditions, in addition to reducing carbon dioxide emissions, this paper mainly analyzes the method of carbon sequestration through the biosphere, using forest to absorb carbon dioxide. However, in social life, forests can not only be

used to absorb carbon dioxide, but also prevent wind and consolidate sand, protect ecological diversity, and provide tourism sites with their own ornamental value. Trees can also provide various products and services for human beings. Therefore, while considering the absorption of carbon dioxide by forest resources, other social values of forests should also be considered in order to better improve the overall social benefits. In this paper, by analyzing the harvest and planting cycle, the most suitable forest management plan is formulated, and the forest resources are artificially adjusted to obtain the maximum social welfare.

## 1.2. Restatement of the research problem

When determining the optimal forest harvesting and planting cycle, it is necessary to consider the forest social value including forest carbon sequestration and their changes over time. To develop a forest management plan, this article answers the following questions

Firstly, the function of the fixed amount of carbon dioxide by forest with respect to time is constructed to construct the carbon sequestration model. Based on the model construction, the optimal forest harvesting and planting cycle were determined, so that the forest model had the highest efficiency of fixing carbon dioxide within a certain period of time.

Secondly, this paper considers the comprehensive value of the forest model, that is, in addition to the fixed amount of carbon dioxide, it comprehensively analyzes the social benefits that multiple forests can provide, such as windbreak and sand fixation, protection of ecological diversity, their own ornamental value and tourism value, and providing the economic value of wood and other products. Based on the comprehensive analysis of multiple indicators, a decision-making model is constructed to optimize the model for the purpose of maximizing the comprehensive value of forests and to give the optimal harvest and planting cycle. In order to apply the model more practically, it is necessary to consider the situation that forests cannot be harvested. Different types of forests are analyzed, the transition conditions of model application are considered, and the adjustment method of application model is determined from the known forest characteristics.

For a certain forest application structure model. Through the empirical analysis of the model, the best management plan determined by the optimization of forest application is planted and harvested. And within 100 years, the carbon dioxide absorbed by the forest was calculated. The data analysis proves that the plan can bring the highest social value. Harvest planting plans to extend the 10-year harvest transition.

## 2. Construction of models

### 2.1. The Model of Carbon Sequestration

Firstly, the carbon sequestration is expressed as a function of area, precipitation and temperature, and the tree species, age ratio and climatic zone are set as important parameters. Among them, the area, precipitation and temperature can be fitted by grey correlation degree based on the collected data, and the corresponding weights are obtained. Considering that area can be expressed as a function of harvest cycle, harvest amount and planting amount, the specific expression of area and time can be sequestration can be obtained by incorporating it into the expression of carbon sequestration. At the same time, by derivating the expression of area to time, we can obtain the harvest period and harvest amount that make the area largest, that is, the maximum carbon sequestration, and then determine the corresponding forest management scheme.

Fitting Single Tree Growth Curve and Forest Area Function

Set the initial forest area as  $S_0$ , trees' growth curve is  $V(t)$

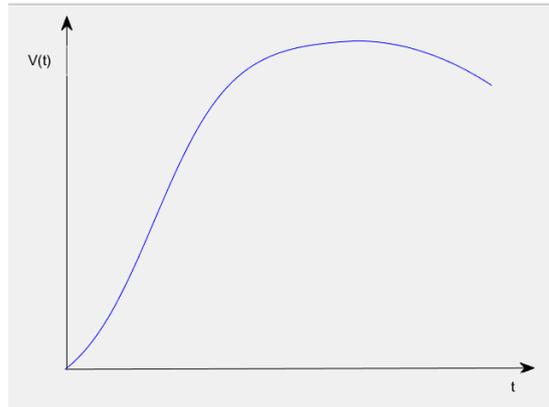


Figure 1: Trees' growing rule

The tree growth curve is obtained by curve fitting without considering the decay of the end period

$$V(t) = \frac{kx_0}{(kx_0)e^{-rt} + x_0} \tag{1}$$

The forest area after planting and harvesting is S

$$S = S_0 + S_P - S_H$$

Then the area after time t is

$$S_1 = S_P \cdot V(t)$$

Consider planting and harvesting, taking into account time changes

$$S(t) = S_0(t) + S_P \cdot V(t) - S_H \cdot \varepsilon(t)$$

Optimization of model

Derivation of time to get the change of forest area to time

$$\frac{dS}{dt} = S'_0(t) + S_P \left[ \frac{dV}{dt} - S_H \cdot \delta(t) \right]$$

$\varepsilon(t)$  is a step function and  $\delta(t)$  is a unit pulse function

Obviously, the derivative cannot be used to seek the maximum carbon sequestration efficiency at this time, and the discretization method is needed to traverse the solution..

Set the planting and harvesting cycle to  $T_0$

If the above process is repeated, we get

$$S(t) = S_0(t) + [S_P \cdot V(t) - S_H \cdot \varepsilon(t)] + [S_P \cdot V(t - T_0) - S_H \cdot \varepsilon(t - T_0)] + [S_P \cdot V(t - 2T_0) - S_H \cdot \varepsilon(t - 2T_0)] + \dots$$

By sorting out the formula, we can get

$$S(t) = S_0(t) + \sum [S_P \cdot V(t - nT_0) - S_H \cdot \varepsilon(t - nT_0)] \tag{2}$$

It can be seen that this is a discrete expression, and it is impossible to obtain a more concise form, nor to determine the size of the carbon sequestration rate by derivation. However, by changing  $S_H$  and  $T_0$ , the size of the carbon sequestration rate can be determined by comparing the amount of carbon sequestration in the same time, so as to obtain the optimal management scheme only considering carbon sequestration.

Construct carbon sequestration model

In general, the target forest can be regarded as a forest without artificial interference ( no harvest ), and its carbon sequestration model is

$$\begin{cases} CS(t) = [A_1 \cdot S(t) + A_2 \cdot w + A_3 \cdot T] \cdot P_1 \cdot P_2 \\ S(t) = S_0 \\ V(t) = \frac{kx_0}{(kx_0)e^{-rt} + x_0}, \quad r > 0 \end{cases}$$

After joining the harvest plan, the harvested wood remains stored carbon, and the carbon sequestration model becomes

$$\begin{cases} CS(t) = [A_1 \cdot S(t) + A_2 \cdot w + A_3 \cdot T] \cdot P_1 \cdot P_2, \quad A_1, A_2, A_3, P_1, P_2 > 0 \\ S(t) = S_0(t) + \sum [S_p \cdot V(t - nT_0) - S_H \cdot \varepsilon(t - nT_0)] + i \cdot S_H \\ V(t) = \frac{kx_0}{(kx_0)e^{-rt} + x_0}, \quad r > 0 \end{cases} \tag{3}$$

$i$  is the total number of harvests within a specified time,  $V(t)$  is the growth curve of a tree species, and Logistic model is adopted.  $S_p$  is the area of newly planted trees each year, and generally the maximum value is taken.  $S_H$  represents the harvest area Assuming that it does not change with time,  $S_0$  is the initial area, which is also consistent with the Logistic model.

### 2.2. The Model od Decision

The forest benefit is quantified as the function of forest storage, water storage, carbon sequestration, oxygen release, soil and sand control benefit, and the fuzzy multi-objective evaluation is carried out. The benefit weight can be determined according to the literature. Among them, each index can be used to determine the expression of carbon sequestration model similar to the first question, that is, the use of grey correlation model for weight values, and finally unified into, you can get the expression of forest benefits. In order to verify whether there is a condition of no-harvest forest, this paper sets the forest harvest to 0, and discusses whether the forest benefit function can obtain the maximum value in the original harvest cycle. If so, it shows that there is a condition of no-harvest forest. In this paper, the transition point is defined as the critical point of increasing forest benefit and decreasing forest benefit. In order to find the transition point, we traverse the harvest area and planting area in a range to find the corresponding area when the maximum value of forest benefit function is taken to the maximum, which is the transition point. For any forest, the transition point of the forest can be found only by giving its climatic zone, age ratio, area and other corresponding parameters into the constructed model.

#### Construct Benefit Evaluation Function

##### Constructing Forest Benefit Evaluation Function

$$Val = B_1FV + B_2WS + B_3CS + B_4QO + B_5BS \tag{4}$$

Let Val be the objective function for the values of  $B_1, B_2, B_3, B_4, B_5$

In the formula:

$$\begin{cases} FV = P_1P_2[k_1S(t) + k_2w + k_3T] \\ WS = P_1P_2[k_4S(t) + k_5w + k_6T] \\ CS = P_1P_2[k_7S(t) + k_8w + k_9T] \\ QO = P_1P_2[k_{10}S(t) + k_{11}w + k_{12}T] \\ BS = P_1P_2[k_{13}S(t) + k_{14}w + k_{15}T] \end{cases}$$

Insert each index equation into the formula and get  $Val = Val(t)$

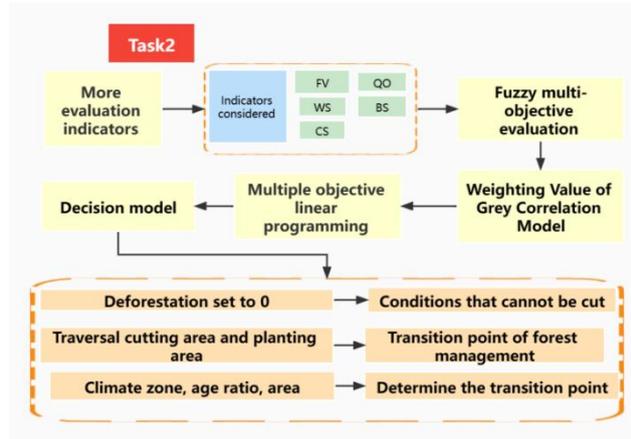


Figure 2: Flowchart of decision model

Considering the condition that forests cannot be harvested

Let  $S_H = 0$ , discuss whether  $Val = Val(t)$  can get maximum in  $0 \leq t \leq T$ . If the maximum value is obtained, the parameter condition for the maximum value is the condition that the forest cannot be harvested. On the contrary, if the maximum value is not obtained, there is no condition that the forest cannot be harvested.

Consideration of transition points applicable to the model

Define transition points as critical points for increasing and decreasing forest benefits

Let  $T_0$  and  $S_H$  traverse a certain range and find  $T_0$  and  $S_H$  corresponding to the maximum value of  $Val = Val(t)$

### 3. Application of model

For the above model, only given the above parameter values and index coefficients, the forest characteristics can be determined. When the time develops to 100 years, the carbon sequestration model in the first question is brought into the corresponding time, and the carbon sequestration in 100 years can be calculated. By repeating the second question, the best management scheme for the forest can be obtained. The strategy related to forest benefit can be substituted into the second question decision model. The harvest period is increased from  $T$  to  $T + 10$ , and then the harvest area is traversed to find the corresponding harvest area that can minimize the change of forest benefit function value, namely the harvest area scheme.

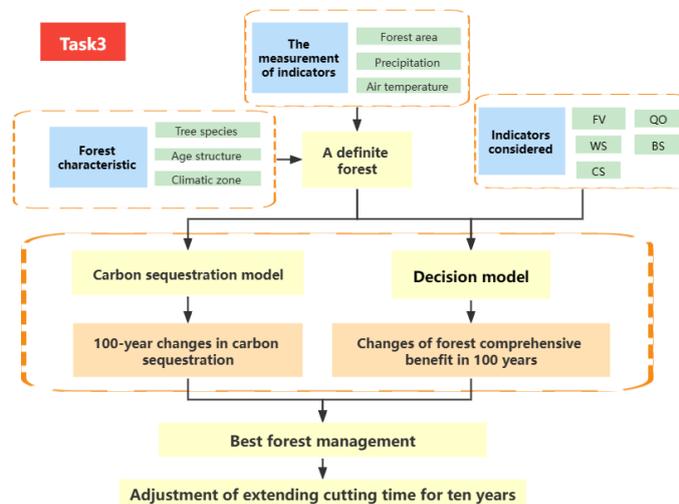


Figure 3: Flowchart of models application

### 3.1. Determine a forest

Our team selected a dam forest system in northwestern China as the research object. Firstly, the relevant data in the past 20 years are solved.

According to the data, spruce is the main tree species, so the carbon sequestration rate of spruce at different forest ages is used to solve  $V(t)$ .

Table 1: Age structure of trees

Young	Middle	Pre-mature	Mature	Over-mature
<40	40-60	60-80	80-120	>120
1.88	3.33	4.61	3.48	3.69

Using logistic model to fit the function, the sum of squares of residuals is 0.9824.

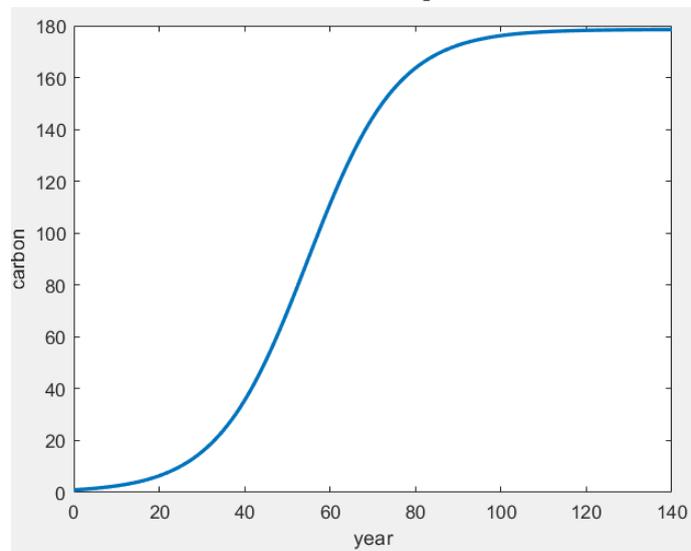


Figure 4:Growth curve of spruce

The expression is

$$V(t) = \frac{C}{e^{-0.0948t} + 0.0056}$$

C is the undetermined coefficient, after normalization, C is  $\frac{7}{1250}$ , namely

$$V(t) = \frac{7}{1250} \cdot \frac{1}{e^{-0.0948t} + 0.0056} \tag{5}$$

given that

$$S(t) = S_0 + \sum [S_p \cdot V(t - nT) - S_H \cdot \varepsilon(t - nT)]$$

$S_0$  is the function of area change under natural condition, which is still fitted by Logistic model.

Expression :

$$S_0 = \frac{91.0104}{0.217e^{-0.0497t} + 0.7149}$$

residual sum of squares is 48.9479

Next, solve the forest benefit evaluation function

First of all, five indexes affecting forest benefit, such as forest volume, water storage, carbon sequestration, oxygen release, soil and sand control benefit ( forest coverage rate ), are considered. According to the paper of Jin Xuetian ( Yanji City Forest Resources Evaluation 2019.5 ), the coefficients of  $Val(t)$  ( unit : billion yuan ) can be obtained.

$$Val(t) = 0.133FV + 6.11WS + 12CS + 0.1QO + BS$$

With forest area, precipitation and temperature as indicators, grey correlation analysis was conducted on forest volume, water storage, carbon sequestration, oxygen release, and soil and sand control benefits ( forest cover rate ), respectively.

The results of correlation coefficient are as follows

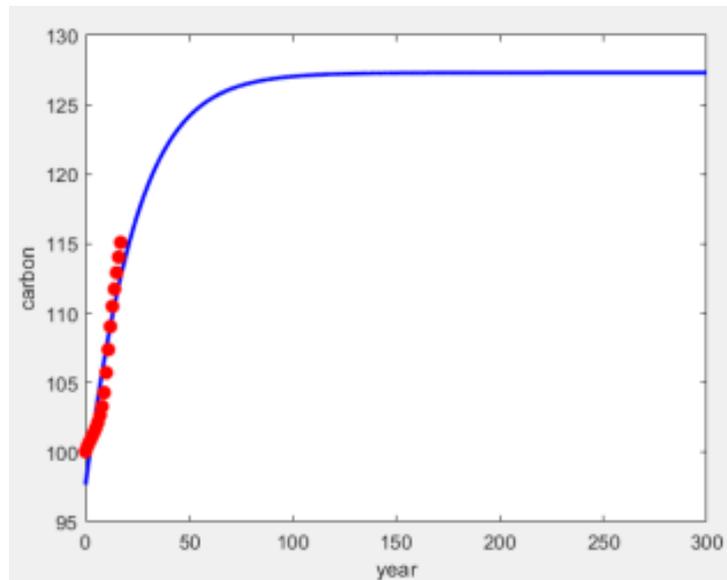


Figure 5: Area of spruce forest

Table 2: Results of Grey Correlation Analysis

Index	Coverage area	Precipitation	Temperature
Forest volume (10,000m <sup>3</sup> )	0.9454	0.7347	0.9177
Water-storing (100millionm <sup>3</sup> )	0.8578	0.7335	0.8362
Carbon sequestration	0.9454	0.7347	0.9177
Quantity of oxygen release	0.9454	0.7347	0.9177
Benefits of Soil Fixation	1	0.7412	0.9598

Taking carbon sequestration as an example for linear fitting, the coefficient can be obtained. By linear fitting these correlation coefficients with the original data, the relevant expressions can be obtained. After integration,

$$Val(t) = 0.133FV + 6.11WS + 12CS + 0.1QO + BS \tag{6}$$

$$\begin{cases} FV = 1.2423[14.7392 \cdot S(t) + 1.214w - 4.096T] \\ WS = 0.037[7.05S(t) + 0.1371w - 2.7447T] \\ CS = 0.0437[5.8234S(t) + 0.9603w - 2.0877T] \\ QO = 0.189[5.590S(t) + 0.9837w - 2.0644T] \\ BS = 0.2756[2.6865S(t) + 1.4037w - 1.407T] \end{cases} \tag{7}$$

And derived from this

$$Val(t) = 7.9594S(t) - 486.7025 + 3.054S'(t) \tag{8}$$

Where S '( t ) is harvested forest area

Then the optimal management plan ( i.e.  $T$  and  $S_H$ ) was solved by traversal, and the maximum annual planting amount was set to 2. The changes of each index in 100 years were calculated. In the figure, the blue solid line represents the change curve of forest state after taking the best management measures ( including harvesting ), and the red dotted line represents the change curve of forest state under natural state.

Area changes are shown below

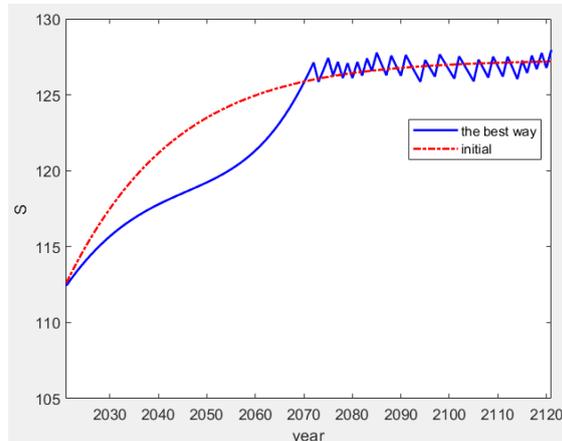


Figure 6: Changes of forest area

According to the carbon sequestration model, the change of carbon sequestration in 100 years is shown as follows :

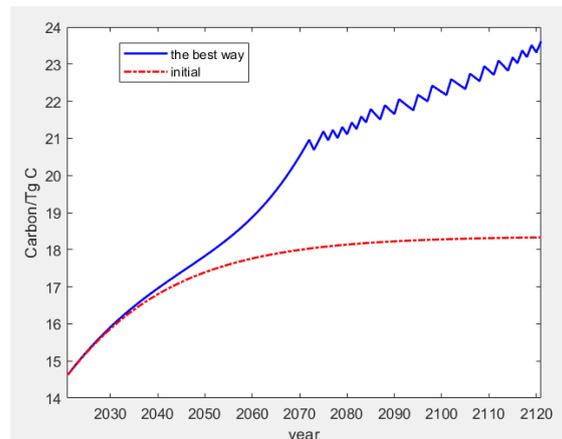


Figure 7: Changes of forest carbon sequestration

According to the decision model, the value changes are shown as follows:

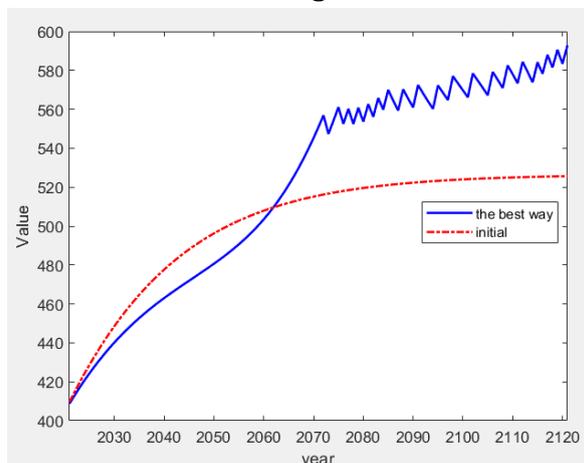


Figure 8: Changes of forest comprehensive value

Average forest benefits for 100 years under the best programme were 511.6935 and 501.4226 under natural conditions

Result analysis : Considering that the selected forest object is the forest that will reach the saturation state, the final result will be affected by the environmental capacity ; the planting and harvesting cycle is relatively short, because there is enough forest reserves.

### 3.2. Consider extending the 10-year harvest

If the harvest time in the programme is extended by 10 years, in order to find the best change programme, it is necessary to separate the planting and harvest cycles, with the objective of minimizing the change in benefits compared with those in the absence of the programme, to traverse the planting cycle and harvest area ( where the planting area remains the maximum ) to obtain the best programme. Taking into account the sensitivity of forest managers and forest users, this paper assumes that they prohibit increasing the intensity of harvesting the original forest,  $T_0$  is 1 year. Taking the data used in this part as an example, it can be obtained that when the planting cycle is shortened by 0.65 times, the harvest cycle is  $T_0$  from the first to the twentieth time, and the twenty-first time, that is, the harvest cycle is increased by 0.2 times each time and the harvest amount is increased by 0.15 times from the beginning of the 21 st year, so that the harvest cycle is prolonged by 10 years after about 33 years compared with the beginning, and the average annual harvest amount changes little, which is about 9.3627 % less than the original benefit.

## 4. Conclusion

we derived a carbon sequestration model using multi-objective programming, which can explain the relationship between carbon sequestration and forest area, tree species, precipitation, temperature, age structure and climate zone. Logistic model is adopted when considering the influence of forest area and tree species. By introducing planting and harvesting period  $T_0$  and harvesting area  $S_H$  into the individual growth curve  $V(t)$ , the general expression of area is obtained.

Since this expression is discrete and related to  $T_0$  and  $S_H$ , the discretization method can be used to traverse  $T_0$  and  $S_H$  to obtain the maximum carbon sequestration stock, corresponding to the optimal management scheme under this condition.

In addition, we first use the multi-objective programming method to get the forest benefit evaluation function  $Val(t)$ , which uses a total of five indicators are forest volume, water storage, carbon sequestration stock, oxygen release, soil and sand control efficiency ( forest coverage ), a comprehensive consideration of the natural to social impact. The best management scheme can be obtained by solving the maximum value of forest benefit by traversal. The management plan proposed by this decision model is suitable for forests without harvesting constraints. In addition to the reasons mentioned above, forests may not be harvested because of historical value and social value. There is a transition point between management plans applicable to all forests, and the transition point is defined as the critical point increase and decrease of forest benefits.

The application of the model shows that the forest and its products will reduce about 32.9882 Tg CO<sub>2</sub> in 100 years. This forest needs to adopt a plan with a planting and harvesting cycle of one year, a planting area of 20,000 mu and a harvesting area of 20,000 mu. is about to reach the environmental capacity, it is suitable for a short planting and harvesting cycle. In addition, this plan has maintained the natural value of forests and certain social value within 100 years. It can be seen from the image that the forest benefit after the plan is greater than that without the plan.

If the harvest time in the programme is extended by 10 years, in order to find the best change programme, it is necessary to separate the planting and harvest cycles, with the objective of minimizing the change in benefits compared with those in the absence of the programme, to traverse the planting cycle and harvest area ( where the planting area remains the maximum ) to obtain the best programme.

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These authors contributed equally to this work and should be considered co-first authors.

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