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Solve the Ideal Paraboloid Equation by 3D Mesh Curvature Algorithm

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Abstract

To solve the problem of the accuracy of fitting the surface of the reflecting surface main rope node compared with that of the ideal paraboloid under different celestial orientations of the FAST radio telescope, a curve integral optimization model is established to solve the shortest stretching stroke, and the three-dimensional surface model is downscaled to obtain the two-dimensional curve model, and the coordinates of the parabolic boundary and the circular arc boundary are obtained. The model is then up-dimensioned to determine the vertex coordinates and focal length of the ideal paraboloid when the optimal solution is obtained, and finally, the equations of the ideal paraboloid are obtained.

Keywords

3D mesh curvature algorithm, curve integral, ideal paraboloid.

1. Introduction

The main project of the FAST radio large telescope started in 2011 and was completed in 2016, is currently the world's largest filled-aperture radio telescope.FAST consists of an active reflecting surface system, feeder support system, measurement, and control system, receiver and terminal and observation base, etc. The completion of FAST has created a new model for building giant telescopes, and its signal reception sensitivity is 2.5 times higher than the second one in the world. Therefore, the study of the structure and working principle of FAST is conducive to greatly expanding human vision and exploring the origin and evolution of the universe.

2. The equation of the ideal paraboloid

2.1. Model Establishment

Let the coordinates of the vertex be (0,m) and obtain the equation of the ideal parabola as $y = \frac{x^2}{2p} + m$.

The radius of the active reflecting surface of the FAST reference state R is about 300 m. From the relative position relationship between the focal plane and the reference sphere F=0.466R, the parabolic focal length:

$$p/2 = -(m + (1 - 0.466)R) \tag{1}$$

Therefore, the equation of the parabola can be determined from the coordinates of the vertex, i.e., the value of m.

The average distance from each main cable node to the center of the circle is calculated to obtain R_1 =300.400 (three decimal places are retained).

Substituting the coordinates of the vertex, we obtain the equation of the parabola when the vertex is at the lowest point of the reference sphere. Extending this to the 3D model, the ideal parabolic equation is initially obtained.

$$z = \frac{x^2 + y^2}{559.2} - 300.4 \tag{2}$$

2.2. Build differential equation optimization model

Due to the constraint actuator tip retraction range of -0.6 m to 0.6 m, then [-299.8,-301] is the distance that the vertex moves in the Z-axis moving distance m interval, within which the profile analysis is performed and the parabola intersects the reference arc, taking -300.4 as the initial value of the vertex. If the vertex moves upward, it causes the whole parabola to be located above the reference arc, which makes the displacement of other main rope nodes in the irradiation range increase, so the vertex can only move downward. Therefore, the goal is to find the minimum curve integral between the parabola and the ideal reference arc during the downward movement of the vertex and find the distance of movement at this time m value. The specific schematic diagram is shown in Figure 1.

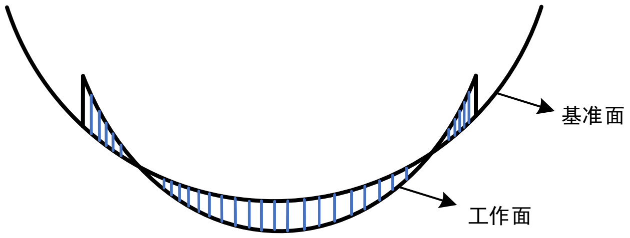

Figure 1 Schematic diagram of integration of circular and parabolic curves

It also minimizes the boundary node movement distance. The search substitutes each m back to get the parabolic equation and finds its smallest m by integrating the area in the middle of the two curves.

Since the maximum change in radial motion of the actuator in the z-axis direction is 0.6m, the search interval was determined to be [-300.4, 301].

When the vertex moves down the parabola and the arc must intersect, the joint is established:

$$\begin{cases} x^2 + y^2 = R^2 \\ y = \dfrac{x^2}{-4(m + (1 - 0.466)R)} + m \end{cases} \tag{3}$$

It is sufficient to consider only the x positive semi-axis and find that the equation must have two real roots on it.

Next, we find the boundary points (x',y') and the intermediate intersection points (x_1,y_1), (x_2,y_2), and then integrate these 3 intervals [0, x_1], [x_1, x_2], [x_2, x'].

By the symmetry of the sphere and the rotating paraboloid, the problem is simplified to consider a segment of arc on a two-dimensional plane moving from the reference plane to the paraboloid, because ignoring the small change in the distance between the main cable nodes before and after the move, the arc length remains unchanged.

The calculated parabolic half-arc length is approximately 156.87m (±0.3). Taking this data as the reference arc length L. According to the formula $L = R\theta$, we can find the angle θ and get the coordinates of the boundary point of the arc (x', y') = ($R\sin\theta$, -$R\cos\theta$).

Integrate and plot the results into the curve, observe that as the transverse coordinate (the distance the vertex moves downward) increases, the area between the two curves keeps getting smaller, and analyze the results to get the following conclusion: the center of the irradiated surface, i.e., the axis of the elliptical paraboloid, is as concave as possible, i.e., the lower tension cable at the boundary point should be stretched as much as possible.

Consider again the constraint as:

1) After the adjustment of the main cable node, the distance between adjacent nodes may change slightly, and the change cannot exceed 0.07%.

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$$-0.07\%\left\|q_i^0 - q_j^0\right\|_2 \le \left\|q_i - q_j\right\|_2 - \left\|q_i^0 - q_j^0\right\|_2 \le 0.07\%\left\|q_i^0 - q_j^0\right\|_2 \tag{4}$$

Among them, $\forall i,j \in Q, i \ne j, Q = \{q_i = (x_i, y_i, z_i) | i \in [1,2226]\}$ is the set of position points of all principal cable nodes, q_i^0 is the position of the ith is the position of the first principal node in the base state, and q_i is the position of the first$i$ is the position of the first principal node in the working state, and q_i with q_j adjacent to; $\left\|q_i - q_j\right\|_2$ is the two-parametric number of any two vectors in the coordinate matrix, i.e., the distance between two points.

2) The actuator expansion is in the positive direction along the radial spherical center of the reference sphere. In the base state, the actuator top radial expansion of the end is 0, and its radial expansion range is [-0.6, 0.6] (unit: m). It is easy to know that the normal vectors of the point q_i in the set Q is $n_{q_i} = (-x_i, -y_i, -z_i)$. Therefore, the coordinates of the main cable node, the upper endpoint of the actuator, and the lower endpoint of the actuator are proportional.

Since the illuminated area has a diameter range of 300m which is equal to the approximate spherical radius, the model can be optimized. Firstly, the boundary of the illuminated area is determined as the line of intersection between the cone and the sphere with the orthogonal view as a square triangle. By processing the data in Annex 1 through Excel, we get the projection point of each main rope node when it is in the XY plane, the distance from the projection point to the coordinate origin is less than 150m. 706 main rope nodes in the illumination area are obtained.

If the lower cable is stretched radially, the coordinates of the intersection of the line are formed by the connection between the origin of the coordinates and the main cable node on the datum and the paraboloid of the ideal ellipsoid can be found. The equation of the line connecting the main cable node on the datum to the origin of the coordinates O is:

$$\frac{x - x_i}{-x_i} = \frac{y - y_i}{-y_i} = \frac{z - z_i}{-z_i}, i \in [1,706] \tag{5}$$

The coordinates of the intersection of the line with the paraboloid of the ideal ellipsoid can be found by combining equations (3) and (5). Let the set of coordinates be the set of points $Q_1 = \{q_m = (x_m, y_m, z_m) | m \in [1,706]\}$.

If we want the working paraboloid to be as close as possible to the ideal paraboloid, we need the main cable node to stretch or contract along the radial direction of the reference sphere to the center of the circle (i.e., the coordinate origin), and during the stretching and contraction, the absolute value distance should be less than or equal to 0.6 m. Let the actual set of point coordinates after stretching or shrinking be the set $Q_2 = \{q_n = (x_n, y_n, z_n) | n \in [1,706]\}$.

Let d_1 be the distance from the reference spherical principal cable node along the radial direction of the sphere to the intersection of the ideal paraboloid and the radial straight line, i.e.

$$D_1 = d_1 = \|Q_1 - Q\|_2 \tag{6}$$

If the distance d_1 is less than $0.6m$ then the actuator connected to the main cable node stretches or contracts the distance d_1; if the distance d_1 is greater than 0.6 m then the actuator connected to the main cable node is stretched or contracted by 0.6 m. To confirm whether the actuator is stretching or contracting, the three coordinates should be differenced and the point set Q minus the set of points Q_1. If the three differences are negative, the actuator should be contracted; if the three differences are positive, the actuator should be stretched.

Finally, the final set of points obtained Q_2 are fitted to the surface by Matlab to obtain the approximate equations of the paraboloid and compared with the ideal paraboloid equations. Combining the point set Q_1 and Q_2 are correspondingly subtracted in turn to obtain the deviation matrix of the actual and ideal positions of the main rope nodes D_1, that is:

$$D_2 = \|Q_1 - Q_2\|_2 = d_{ij}, \forall i,j \in [1,706], i = j \tag{7}$$

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Subsequently, the matrix is variance-processed and its minimum value is found under the constraints given in the question. Let the matrix D_1 the overall mean of $\bar{d}_{ij}$ and the overall sample size N is 706. i.e., the objective function:

$$\min Z_1 = \sigma_1^2 = \sum \left(d_{ij} - \bar{d}_{ij}\right)^2 / 706 \tag{8}$$

2.3. Optimization of the model

The above model description does not fit the actual surface well to the ideal parabolic surface. Since the radial distance between two points is directly used as the basis for stretching or contracting a certain distance, it may result in a lower similarity and bonding as well as a higher amount of deviation between the actual surface and the ideal parabolic surface. Therefore, the concepts of curvature and radius of curvature are introduced. The variance between the radius of curvature of the actual point and the curvature of the corresponding point on the ideal paraboloid surface is used as the criterion for the degree of fit.

The vertices of the triangular mesh are discrete and we can only use the mesh discrete curvature algorithm.

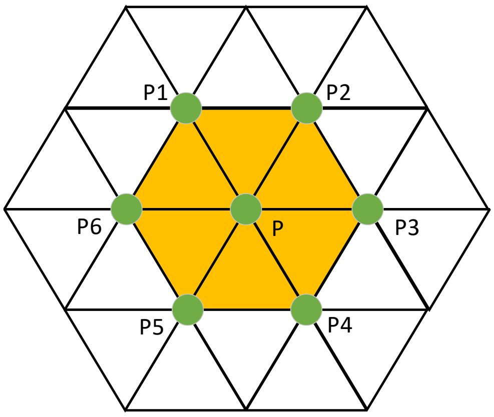

Figure 2 Triangular grid plan diagram

As in Figure 7, for each vertex on the grid p, take the first-order neighborhood of p (orange range B), and define the following matrix.

$$E_p(B) = \frac{1}{|B|} \sum_{e \in B} \beta(e) \|e \cap B\| \bar{e}^T \cdot \bar{e} \tag{9}$$

Among them, the $|B|$ is the area of B; e is the edges of the first-order neighborhood polygon region of the orange p in the grid vertex B; $\bar{e}$ is the unit vector in the direction of e; $\|e \cap B\|$ is the length of $e \cap B$, the polygon that always lies between 0 and $|e|$; $\beta(e)$ is the angle between two normal triangles with e is the angle between two normal triangles with common sides.

Analyzing the eigenvalues and eigenvectors of the above matrix, in the Hessian matrix $E_p(B)$ in the Hessian matrix, the minimum eigenvalues and eigenvectors can be used as the normal vector estimators for the vertex p. The principal curvature, mean curvature, and Gaussian curvature of the mesh surface are calculated.

The set of points Q_1 and Q_2 into the triangular grid discrete curvature algorithm to obtain the curvature matrix $P_1 = \{\rho_i | 1 \le i \le 706\}$ and $P_2 = \{\rho_j | 1 \le j \le 706\}$. Subsequently, the two matrices are subjected to a subtraction operation to obtain the actual and ideal deviation one-way volume matrix$D_2 = \rho_{ij}$, so that the overall sample mean is$\bar{\rho}_{ij}$, the variance of the actual and theoretical quantities, i.e., the standard deviation of the finite number of measurements, is found by the mean and var functions in MATLAB. And let this deviation obtain the minimum value under the constraint. That is the objective function.

$$\min Z_2 = \sigma_2^2 = \sum \left(\rho_{ij} - \bar{\rho}_{ij}\right)^2 / 706 \tag{10}$$

2.4. Solving the model

First, the ideal paraboloid when the actual situation such as actuator contraction is not considered is plotted and shown in Figure 3. As seen in Figure 3, the top view of the ideal paraboloid is square. This is to facilitate the display of the degree of curvature of the paraboloid (i.e. curvature).

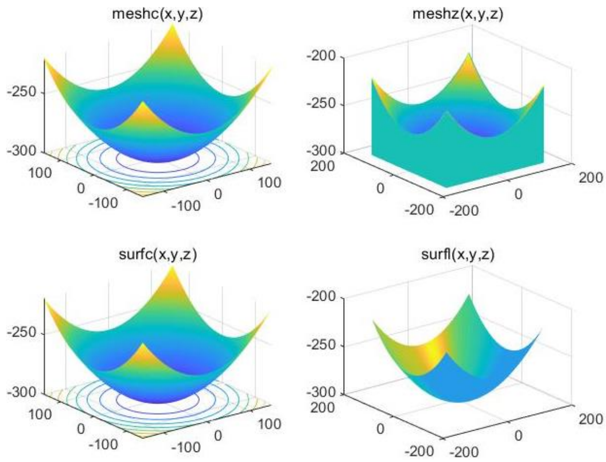

Figure 3 Ideal paraboloidal three-dimensional diagram

Second, the actual situation such as the actuator expansion displacement limit and the small change of distance between the main cable nodes after the expansion is considered. For the vertex motion distance limitation, the step size is taken as 0.01 for the search accuracy, and the coordinates of the two boundary intersection points are obtained by associating the original parabolic equation and the spherical equation. The relationship between the amount of vertex displacement change and the magnitude of the curve integral between the two curves is plotted as shown in Figure 4. And the variation of the paraboloid-sphere deviation distance with the focal length of the paraboloid is shown in Figure 5.

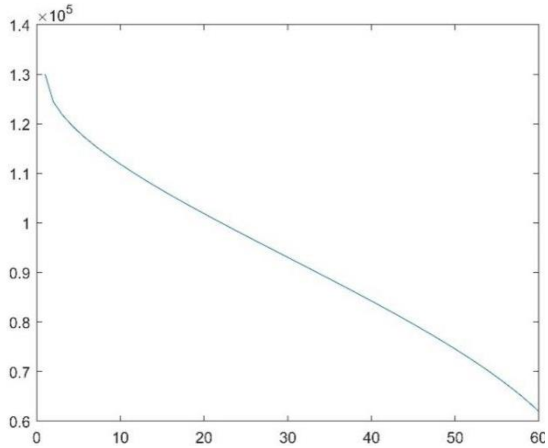

Figure 4 Vertex offset and a boundary error

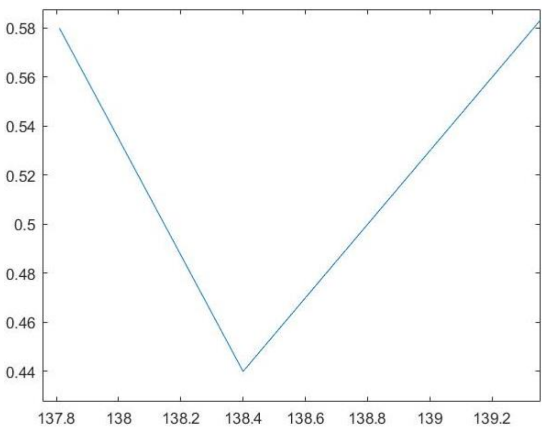

Figure 5 Distance between parabolic vertices and spherical vertices corresponding to different focal lengths

From the above two figures, it can be observed that when *m* is taken as -0.6, the area integral between the two curves is minimized; when the focal length is taken to be approximately equal to 138.4, the offset between the paraboloid and the surface is minimized. That is, the final result is that the paraboloidal vertex is at (0,0,-301) When the focal length is 138.4, the deviation is minimized, and the optimal result is obtained at this time. The coordinates of the parabolic boundary are calculated as (150,-260.2694) and the arc boundary coordinates are (149.8586,-260.3508), the ideal parabolic equation after considering the actual situation is:

$$z = \frac{x^2 + y^2}{553.6} - 301 \tag{11}$$

By verification, the boundary node motion distance is 0.1632 m when the vertex vertical coordinate is -301 satisfying the constraint condition. The scatter cloud fit of the actual reference circular surface after the actuator movement for each main cable node in the follow-up processing step is shown in Figure 6. the objective function Z_1 = 1.4872 and Z_2 =8.4907. The contour map corresponding is shown in Figure 12.

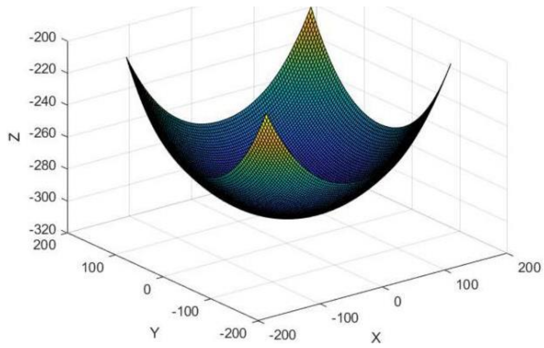

Figure 6 Scatter fit surface of main cable node after adjustment in 150 m range

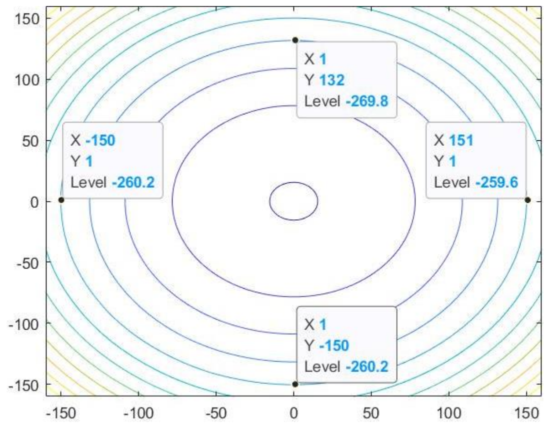

Figure 7 Contour map of the scatter-fitted surface of the main cable node after adjustment within 150 m

3. Conclusion

For solving the ideal parabolic equation model, this paper has well combined the constraints to limit the model so that the results are within a reasonable limit and have some practical applications.

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