ISSN: 2664-9640

Construction and Application of MCSO Model for Determination of Crop Income Insurance Rate

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Abstract

Agriculture is the foundation of China's national economy. As the core of agriculture, crop production is related to the country's destiny. Crop insurance can effectively reduce the risk of agricultural product planting and escort agricultural development. This paper takes the determination of crop income insurance rate as the research object. In order to obtain the best insurance rate, a mixed Copula-stochastic optimization model (hereinafter referred to as the MCSO model) is established. The objective function is established with the lowest risk of farmers' planting income, the insured amount and rate are used as decision variables, and farmers' income risk, insurance company earnings expectations and insurance premium limit are constraints. In the algorithm design, the mixed Copula method and the EM algorithm are used to fit the joint distribution function of agricultural output and price. Finally, taking corn as an example, the stochastic optimization method is used to solve the problem, and the insured amount is 1124.16 yuan per mu, and the insurance rate is 2.24%, which verifies the practicability of the model.

Keywords

Crop income; Premium rate; Mcso model; EM algorithm; Corn.

1. Introduction

Crop insurance is currently one of the means to solve the issues relating to agriculture, rural areas and farmers. Among them, agriculture is the foundation of the national economy, but it is also a weak industry due to its long production cycle, large environmental impact, and slow returns [1]. Therefore, the development of agriculture requires policy-based agricultural insurance to escort it. Agricultural insurance is an effective agricultural risk avoidance and apportionment mechanism, and one of the core methods of modern agricultural risk management. It belongs to the "GreenBox Policies"[2] and is one of the effective tools for countries to promote agricultural development. Judging from international experience and the current situation, agricultural income insurance may be an important means of reforming the pricing mechanism of crop products in the future, and it will become an important measure for the state to regulate agricultural development [3]. However, the determination of insurance rates is a difficult issue in developing high-quality and sustainable agricultural insurance [4]. The premium rate is the proportion of the premium charged by the insurer to the policyholder or the insured based on the insurance amount, and is the basis for calculating the premium [5]. Reasonable determination of agricultural insurance rates and protection of farmers' risks are important prerequisites for ensuring the stability of agricultural insurance operations [6]. Nowadays, policy-based agricultural insurance is still developing rapidly, and it is of great practical significance to develop more suitable crop insurance types according to the situation in our country.

The risks faced by farmers are mainly divided into yield risk and income risk [7], so crop insurance can be divided into yield insurance and income insurance accordingly. China's current agricultural insurance system is mainly based on yield insurance, but yield insurance can only guarantee farmers' yield losses in the event of natural disasters, and cannot guarantee the reduction of income caused by the drop in agricultural product prices. Crop income insurance can simultaneously protect farmers from losses in yield and income [8], which makes the pricing of crop income insurance the focus of research on agricultural insurance in recent years. The calculation process of the crop income insurance rate is: calculate the joint distribution of price and yield, and then calculate the insurance rate according to the joint distribution. In 2018, Chao [9] used the binary Copula function to establish the joint distribution of unit yield and price risk , and then used Monte Carlo random sampling to obtain the series of farmers' income distribution, and finally calculated the cotton insurance premium in Xinjiang. In 2021, Wen [10] also used the binary Copula function and the Monte Carlo method to measure the premium of peanuts. In the same year, Zhan [11] made some improvements to the Monte Carlo method to measure the premium of corn. The mainstream rate determination method of income insurance is still the Monte Carlo stochastic simulation method, which is simple, and wide in scope.

However, it is difficult to connect with real-time policies, farmers' expectations of income, and insurance companies' expectations of profitability, resulting in poor flexibility. Therefore, this paper introduces a stochastic optimization model to optimize the method of determining the income insurance rate. Stochastic optimization is an optimization method that introduces random variables into an optimization model, and its main purpose is to solve optimization problems with random variables. This method is mostly used in power systems [12]. There are two main methods for its solution. One is to convert it into a deterministic problem by using the characteristics of the expected value of random variables, and the other is to consider the most optimization problem in the sense probability [13]. This paper chooses the first method. In the process of determining the rate, it is difficult to construct the joint distribution of output and price. The development of Copula function has solved this problem. Copula functions are functions that connect the joint distribution of multiple random variables with their respective marginal distributions. In 1959, Sklar [14] first proposed the Copula theory, and in 2002, Elizabeth [15] applied it to the field of crop yield insurance. The research of Tejeda H A [16] in 2008 showed that there is a negative correlation between agricultural product price risk and yield risk. In 2014, Hui Zeng et al. [17] used the normal coupled Copula function to fit the joint distribution of yield and price. In 2014, Goodwin et al. [18] used the mixed Copula model to analyze the historical data of agricultural products and found that the Vine-Copula model could more accurately estimate the joint distribution of yield and price. In 2011, Woodard J D et al. [19] calculated the joint distribution of output and price by mixing Copula functions, and believed that the result was better than a single Copula function.

From the literature reviewed so far, stochastic optimization models are rarely used in the determination of insurance rates. Therefore, the main purpose of this paper is to construct a mixed Copula function-stochastic optimization model for the determination of agricultural income insurance rates, and to study the algorithm.

ISSN: 2664-9640

2. MCSO Model Establishment and Solution

2.1. Model Assumptions.

In order to solve the problem, we propose the following assumptions:

(i) Farmers have no other source of income other than crop. The research in this paper is crop income insurance, so the income of farmers in other fields is not considered, and only farmers' income from crops is considered in the measurement of farmers' risks.

(ii) It is assumed that there will be no major changes in crop varieties in a short period of time, that is, the annual yield distribution function of crops is the same. Once a major change is made in the variety of crops, such as a sudden increase in output, it will have a greater impact on the yield and price of crops. Mutation of crop varieties is difficult and it will take a certain amount of time to popularize. Therefore, in order to simplify the problem, this paper does not consider Situation in which crop varieties have made great changes in the short term.

(iii) Assuming that there is no fraudulent behavior, that is, maliciously destroying crops after applying for insurance to defraud insurance companies of premiums. Because "insurance fraud" is difficult to define and different insurance companies have different policies, this paper assumes that all farmers do not have malicious insurance fraud.

2.2. Choose Decision Variables.

In the MCSO model of this paper, the unknowns are the insurance amount and the insurance rate, which are also the values determined by the MCSO model. Therefore, this paper selects the insurance amount PI and the insurance rate PR as the decision variables.

2.3. Determine the Objective Function.

This paper has two main objectives, the main optimization objective is to minimize the risk of crops, and the secondary optimization objective is to maximize the average net income of farmers. Among the two optimization objectives, the main optimization objective has a higher priority, so this paper sets it as the objective function of the model. At the same time, in order to achieve the suboptimal objective, it is limited as a constraint condition. It can be obtained that the objective function of the MCSO model is that the risk of farmers after insured is the smallest.

To minimize the risk of farmers as the main optimization goal, it is necessary to define the risk first. The risk is due to the uncertainty of the two random variables which are crop yield and price. The stronger the volatility of these two random variables, the greater the risk that represents the crop. The degree of fluctuation of random variables can be measured by a certain variance, so this paper uses the variance of crop income to represent the risk of farmers.

To solve the risk of farmers after insured, it is necessary to first solve the net income of farmers after insured. The process is shown below.

First, the income and expenditure of farmers after insured are analyzed. The income of farmers after insured includes the amount of crops sold and the insurance company's compensation, and the expenditure includes the fixed expenditure M for planting crops and the premium P paid by farmers. Then the annual net income I of the farmer is:

$$I = xy + C - P - M.$$

In the formula, x is a random variable, representing the yield of crops; y is also a random variable, representing the price of crops. C is compensation. The compensation method is as follows: when the amount of crops sold is lower than the insured amount, the difference between the insured amount and the amount of crop income will be paid, otherwise no compensation will be paid. which is:

$$C = \begin{cases} PI - xy, & xy < PI \\ 0, & xy \geq PI \end{cases}.$$

Solving the average net income LI after insured, we can get:

$$LI = E(xy + C) - P - M$$
$$= \iint (xy + C)f(x, y)dxdy - P - M.$$

After obtaining the net income of the farmers after the insured, the post-insurance risks for farmers is solved. From the basic statistical knowledge, it can be known that the post-insurance risks for farmers LS is:

$$LS = E\left(\left(xy + C - P - M - E(xy + C - P - M)\right)^2\right)$$
$$= E\left(\left(xy + C - E(xy + C)\right)^2\right)$$
$$= E((xy + C)^2) - E(xy + C)^2.$$

The post-insurance risks for farmers is:

$$LS = \iint (xy + C)^2 f(x, y)dxd - E(xy + C)^2$$
$$= \iint (xy + C)^2 f(x, y)dxdy - \left(\iint (xy + C) * f(x, y)dxdy\right)^2.$$

Then the objective function of the MCSO model can be obtained as:

$$min\ LS = \iint (xy + C)^2 f(x, y)dxdy - \left(\iint (xy + C) * f(x, y)dxdy\right)^2.$$

2.4. Establish Constraint.

There are three basic constraints in the MCSO model, namely risk constraint, income constraint and premium constraint.

2.4.1 Risk Restraint.

The post-insurance risks for farmers is lower than The pre-insurance risks, so as to establish constraint 1:

$$LS < FS.$$

FS in the formula is the pre-insurance risk of farmers.

To solve the pre-insurance risk of farmers, the income of farmers is the amount of crops sold, and the expenditure is the fixed expenditure for planting crops M for planting crops. Then the annual net income I of the farmer is:

$$I = xy - M.$$

The derivation process of the formula for solving the pre-insurance risk FS of farmers is given:

$$FS = E((xy - M - E(xy - M))^2)$$
$$= E((xy - E(xy)^2)$$
$$= E((xy)^2) - E(xy)^2.$$

The pre-insurance risk of farmers is:

$$FS = \iint (xy)^2 f(x, y)dxdy - (\iint xyf(x, y)dxdy)^2.$$

It can be seen that the constraint 1 can be expressed as

$$\iint (xy + C)^2 f(x, y)dxdy - \left(\iint (xy + C) * f(x, y)dxdy\right)^2$$
$$< \iint (xy)^2 f(x, y)dxdy - \left(\iint xyf(x, y)dxdy\right)^2.$$

2.4.2 Profit Constraint.

Insurance companies pursue interests. If the insurance company's income is negative, it may cause the agricultural insurance industry to shrink, develop stagnation, and the insurance company will withdraw from the agricultural insurance market. Therefore, it is necessary to ensure that the insurance company's income is positive, so as to establish constraint 2:

$$\iint (P - C) * f(x, y)dxdy \geq 0.$$

2.4.3. Premium Constraint.

Excessive premiums will reduce the enthusiasm of farmers. At the same time, in order to ensure the highest average net income of farmers, it is necessary to limit the premiums charged by insurance companies. The limit of premiums can be obtained through policy requirements, previous income insurance premiums, and questionnaires. The premium amount is set to be less than the constant MA, and the constraint 3 is established as follows:

$$PI \times PR < MA.$$

In the formula, PI is the insured amount of the insurance, and PR is the insurance premium rate.

2.4.4. Other Constraints.

The above three constraints are the necessary basic constraints for the establishment of the MCSO model. One of the advantages of the MCSO model compared to the traditional rate calculation method is that it can limit multiple variables in the process of insurance rate determination.

The variables that can be limited here mainly include the income of crops, the income of insurance companies, the risks of farmers, the price of crops, the amount of insurance, the premium, and the premium rate. Two examples of constraints that may appear in actual situations are given here, and specific constraints can be added according to the actual situation.

(i) A certain place supports agricultural insurance and gives certain subsidy to the insurance company, at this time, the income of the insurance company can be restricted.

(ii) The policy of a certain place re- quires that the premium of income insurance shall not be higher than a certain value. At this time, we can also limit the final premium charged.

2.5. Final Optimization Model.

$$\min LS = \iint (xy + C)^2 f(x, y)dxdy - (\iint (xy + C) * f(x, y)dxdy)^2$$

$$S.T\begin{cases} LS < FS \\ \iint (P - C) * f(x, y)dxdy \geq 0 \\ PI \times PR < MA \\ Others \end{cases}.$$

3. Algorithm solution of MCSO Model

3.1. Research Ideas.

The algorithm solution of the MCSO model is mainly divided into two parts. The first is to solve the joint distribution function of crop yield and price. In this paper, the mixed Copula function is used to construct the joint distribution function fitting model, and the EM algorithm is used to solve the fitting model. Finally, solve the established stochastic optimization model.

3.2. Solving the Joint Distribution Function of Crops by a Single Copula

3.2.1 Marginal Distribution Function Fitting.

Before using the Copula function to solve the joint distribution function of crops, it is necessary to fit the marginal distribution functions of crop yield and price. In the process of fitting the marginal distribution function, different fitting methods can be selected according to the distribution characteristics of the data, such as normal distribution fitting, kernel distribution fitting, etc.

3.2.2 Introduction to Copula Function.

Before fitting the Copula function, the commonly used Copula functions are first introduced. The Copula function is a function that can express the correlation between multiple variables according to Sklar's theorem. The three commonly used and representative Copula functions are: Gumbel Copula function, Clayton Copula function, Frank Copula function [20]. The probability density functions of these three Copula functions are as follows.

(i) Gumbel Copula function

$$f_G(x,y;\theta_G)=\frac{F_G(x,y,\theta_G)(\ln x\cdot \ln y)^{\theta_G-1}}{xy[(-\ln x)^{\theta_G}+(-\ln y)^{\theta_G}]^{2-\frac{1}{\theta_G}}}*\left\{[(-\ln x)^{\theta_G}+(-\ln y)^{\theta_G}]^{\frac{1}{\theta_G}}+\theta_G-1\right\}.$$

In the above formula, $F_G(x,y,\theta_G)=e^{-\{(-\ln x)^{\theta_G}+(-\ln y)^{\theta_G}\}^{1/\theta_G}}$.

(ii) Clayton Copula function

$$f_c(x,y,\theta_C)=(1+\theta_C)(xy)^{-\theta_C-1}(x^{-\theta_C}+y^{-\theta_C}-1)^{\frac{-1}{\theta_C}-2}.$$

(iii) Frank Copula function

$$f_F(x,y;\theta_F)=\frac{\theta_F(1-e^{-\theta_F})e^{-\theta_F(x+y)}}{[(1-e^{-\theta_F})-(1-e^{-x\theta_F})(1-e^{-y\theta_F})]^2}.$$

In the formula, x and y are random variables, θ_G, θ_C and θ_F are the dependent parameters of the Gumbel Copula function, the Clayton Copula function, and the Frank Copula function, respectively.

Different Copula functions have different properties. For example, the distribution of Frank Copula functions is symmetric, while the distributions of Gumbel Copula functions and Clayton Copula functions are asymmetric. The Gumbel Copula function distribution emphasizes the stronger upper tail correlation between random variables, and the Clayton Copula function distribution emphasizes the stronger lower tail correlation between random variables [21].

The fitting of the Copula function generally uses the maximum likelihood estimation method. When only a single Copula function is used to fit the data, there will be problems that the characteristics of the data cannot be described. Therefore, this paper uses the idea of network [22-23] to mix Copula functions with different properties, and use it as a new function. Fitting is performed, which can take into account the properties of different Copula functions to improve the accuracy of data fitting.

3.3. Solution Based on EM Algorithm Mixed Copula Function.

The mixed Copula function is a function obtained by linearly weighting a single Copula function. Assuming that the density function of a single Copula is $f_k(x,y;\theta_k)$, the mixed Copula function is:

$$f(x,y)=\sum_k w_k f_k(x,y;\theta_k).$$

In the formula, w_k is the weight of the kth Copula function.

In the mixed Copula function, parameter estimation of θ_k and w_k is required. Since the maximum likelihood function of the mixed Copula is very complex, this paper uses the EM algorithm [24] to estimate the Copula dependence parameter θ and weight parameter w. The EM algorithm mainly uses the idea of maximum likelihood estimation and Kirchhoff index [25-28]. The main steps of the algorithm are as follows.

Step1: Given the initial value of the dependent parameter θ and the weight parameter w.

Step2: Enter the E step of the EM algorithm. Step E is to calculate the conditional expectation distribution of the log-likelihood function on the unknown data according to the estimated values of the current parameters θ and w.

Step3: Enter the M step, and find the parameters θ and w when the maximum value of the conditional expectation obtained in the E step is obtained.

Step4: When the change of the dependent parameter θ and the weight parameter λ is less than a small number, it means that the algorithm reaches the local optimum, and the algorithm ends to get the parameters, otherwise, go back to Step 2.

We use a mixed Copula model to construct a joint probability density function $f(x,y)$ of crop yields and prices. Assume that the two-dimensional random variables (the yield and price of crops) are (X,Y) respectively, and their marginal distributions are $X\sim f_X(x;\alpha)$, $Y\sim f_Y(y;\ \beta)$, and their mixed Copula function is:

$$f(X,Y;\Phi)=\sum_{k=\{G,C,F\}} w_k f_k(x,y;\theta_k).$$

In the formula, $\theta=(\theta_G,\theta_C,\theta_F)$, $w=(w_G,w_C,w_F)$, $w_G+w_C+w_F=1$. Estimate the parameters w_k and θ_k in the model to achieve the best fitting effect. The joint distribution of (X,Y) is:

$$f(X,Y;\Phi)=\sum_{k=\{G,C,F\}} \lambda_k f_k(F_X(x;\alpha),F_Y(y;\ \beta);\ \theta_k).$$

In the formula, $\Phi=(\alpha^T,\beta^T,w^T,\theta^T)^T$ and $\phi=(w^T,\theta^T)^T$. Then it can be known that the probability density function of crop yield and price (X,Y) is:

$$f(x,y;\Phi)=f_X(x;\alpha)f_Y(y;\beta)*\sum_{k=\{G,C,F\}} \lambda_k f_k(F_X(x;\alpha),F_Y(y;\beta);\ \theta_k).$$

Referring to Daeyoung Kim et al. [29], the EM algorithm is used to estimate the parameters of the mixed Copula model. The EM algorithm mainly uses the idea of maximum likelihood estimation and is an iterative algorithm for solving complex maximum likelihood estimation. First, need to find the maximum likelihood function of (X,Y), which is:

$$L(\Phi)=\sum_{t=1}^{T}[\ln f_X(x_t;\alpha)+\ln f_Y(y_t;\beta)]+\sum_{t=1}^{T}\ln\left[\sum_{k=1}^{M} w_k c_k(F_X(x_t;\alpha),F_Y(y_t;\beta);\theta_k)\right].$$

In the formula, (x_k,y_k) are samples from (X,Y) and M is the sample size.

Then the penalized likelihood function is constructed as:

$$P(\Phi)=L(\Phi)-\mathrm{T}\sum_{k=1}^{M} p_\gamma(w_k). \tag{3.1}$$

The penalty function $p_\gamma(\lambda)$ selects the SCAD penalty function, namely:

$$p_\gamma(w)=\gamma I(w\le\gamma)+\frac{(a\gamma-w)_+}{(a-1)}\,I(w>\gamma)\ (a>2,w>0).$$

Estimate the unknown parameters in (3.1). Using nonparametric kernel density estimation for $\sum_{k=1}^{M}[\ln f_X(x_k;\alpha)+\ln f_Y(y_k;\beta)]$, the estimated value of the density function $\hat{f}_X(x)$ and $\hat{f}_Y(y)$, and then the estimated value of the marginal distribution function $\hat{x}$ and $\hat{y}$ can be obtained.

Put $\hat{x}$ and $\hat{y}$ into (3.1), we can get:

$$\begin{aligned}P(\Phi)=&\sum_{t=1}^{T}\ln[\sum\nolimits_{k=1}^{M} w_k c_k(\hat{x}_t,\hat{y}_t;\theta_k)]\\&-T\sum_{k=1}^{M} p_{\gamma_T}(w_k)\end{aligned} \tag{3.2}$$

Then use the EM algorithm to solve the parameter estimation of equation (3.2). Each iteration of the EM algorithm includes E and M steps. Step E is to calculate the conditional expectation distribution of the log-likelihood function of the complete data with respect to the unknown data based on the existing data (X, Y) and the current parameter estimates. The M step is the parameter when the expected distribution is maximized. Initially, set the parameter estimation value as the fitting function value $\Phi_0 = (\theta_0, w_0)$ of a single Copula. Assuming that the estimated value is $\Phi_k = (\theta_k, w_k)$ in the $k+1$ th iteration, The $k+1$ th estimated value $\Phi_{k+1} = (\theta_{k+1}, w_{k+1})$ is obtained through E and M steps. Stop when $\|\Phi_{k+1} - \Phi_k\| < eps$, eps is a set small number.

Through the EM algorithm, the estimated value of the weight parameter $w = (w_G, w_C, w_F)^T$ and the dependent parameter $\theta = (\theta_G, \theta_C, \theta_F)^T$ can be obtained. Finally, the probability density function $f(x, y)$ of the mixed Copula can be obtained and used to build the MCSO model.

3.4. Solution of Stochastic Optimization Model Algorithm.

The algorithm solving steps of the stochastic optimization model are as follows.

Step1: First, generate random numbers of 100,000 groups of crop yields and prices according to the probability density function $f(x, y)$ of the mixed Copula, and multiply the corresponding crop yield values of 100,000 cases.

Step2: Use the obtained crop income value of 100,000 cases as an approximation of the crop income probability density function, instead of the income probability density function, to solve the income and pre-insurance risk of farmers.

Step3: Give the initial value of the insured amount and the rate to solve the average net income and risk after the crops are insured. For the solution process, please refer to 2.3 above.

Step4: Then solve the objective function and constraints according to the established optimization model. The objective function is the post-insurance of farmers. In risk constraint, the risk before and after the farmer insured has been obtained in Step 2, and the income of the insurance company in income constraint can be calculated by the difference between the income before and after the farmer's insurance. In premium constraint, the farmer's premium is less than constant MA, here MA can be selected according to the actual situation.

Step5: Finally, use interior-point, an algorithm that can solve linear programming and nonlinear convex optimization, to iterate through internal traversal of feasible regions to solve the insurance amount and rate when the risk of farmers is minimized.

4. Application of MCSO Model

4.1. Data Selection and Processing.

In terms of data selection, this paper selects hundreds of sets of corn yield and price data in Baoding, Hebei Province and surrounding areas such as Shunping, Anxin, and Dingxing. Due to the non-uniform dimensions of corn per unit yield and price data, in order to optimize the Copula fitting results, the corn per unit yield and price data are standardized. Taking the unit yield as an example, the standardized processing method selected in this paper is the following formula:

$$x = \frac{x - Ex}{Sx}.$$

In the formula, x is the yield sequence that needs to be standardized, Ex is the arithmetic mean of the sequence, and Sx is the standard deviation of the sequence.

4.2. Marginal Probability Density Function Fitting.

To solve the joint probability density function of corn yield and price, the marginal probability density function of corn yield and price needs to be solved first. Taking yield as an example, in

ISSN: 2664-9640

the process of solving the probability density function of yield, we used the chi-square test method. Respectively, for the cases where the probability density function is Beta, Logistic, Gen. Extreme Value, Weibull, Uniform and other common continuous distribution functions fit the data below. After comparison, the function with the highest goodness of fit was selected as the probability density function of maize yield.

4.2.1 The Solution of The Probability Density Function of Maize Yield.

After many comparisons, the Gen. Extreme Value function with the highest goodness of fit was selected as the probability density function of maize per unit yield. The probability density function of this function is:

$$f(x)=\frac{1}{\sigma}\exp(-(1+kz)^{-\frac{1}{k}})(1+kz)^{-1-1/k}.$$

$$z=\frac{x-\mu}{\sigma}.$$

In the formula, the values of k,σ,μ are 0.01603, 0.8212, -0.48722, respectively.

4.2.2 Solution of the probability density function of corn price.

After many comparisons, the Weibull function with the highest goodness of fit is selected as the probability density function of corn price. The probability density function of this function is:

$$f(y)=\frac{a}{\beta}\left(\frac{y-\gamma}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{y-\gamma}{\beta}\right)^{\alpha}\right).$$

The values of σ,β,γare2.1952×10^{7}, 1.6606×10^{7}, -1.6606×10^{7}. The fitted graph of corn yield and price is shown in Figure 1.

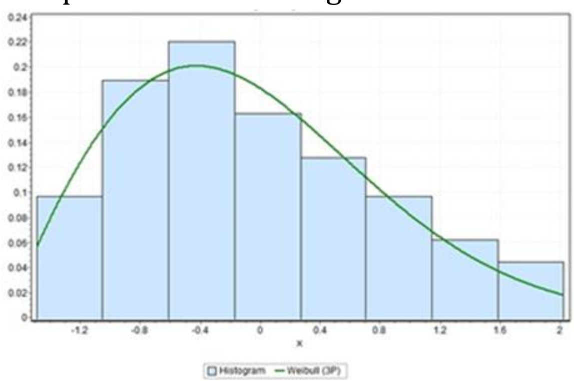

FIGURE 1(A): Fitted graph of corn yield

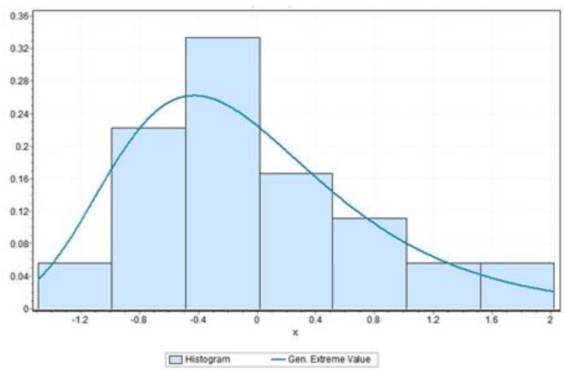

FIGURE 1(B) Fitting diagram of corn price

4.3. Fitting a joint probability density function with a single Copula function.

After solving the marginal probability density functions of corn yield and price, we use Gumbel Copula function, Clayton Copula function, and Frank Copula function to solve the joint probability density function of corn yield and price respectively.

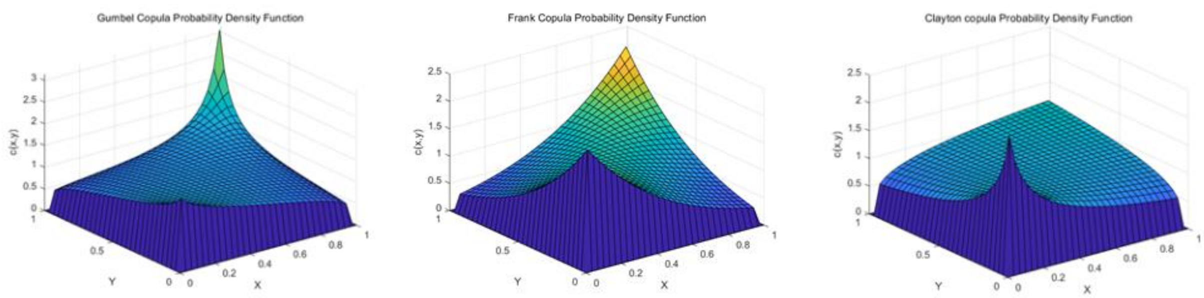

FIGURE 2: Fitting results of a single Copula function

After solving, the dependent parameters of the three Copula functions are 1.3740, 1.3165, and 4.3222, respectively. The fitting result is shown in Figure 2 below.

4.4. Mixed Copula Function Fitting Joint Probability Density Function.

The EM algorithm is used to iterate the dependent parameters and weights of the binary Gumbel-Copula function, the binary Clayton-Copula function, and the binary Frank-Copula function to make it fit the joint distribution function of corn yield and price optimally. The parameter results are shown in Table 1.

TABLE 1: Mixed Copula Function Fitting Results

	Gumbel Copula	Clayton Copula	Frank Copula
Parameters after iteration	0.98814	1.57627	9.28297
weight after iteration	0.59986	0.00000	0.40014

Finally, the mixed Copula function expression is obtained as

$$F = 0.59986C_C(X,Y;0.98814) + 0.40014C_F(X,Y;9.28297).$$

The fitting result of the mixed Copula function is shown in Figure 3 below.

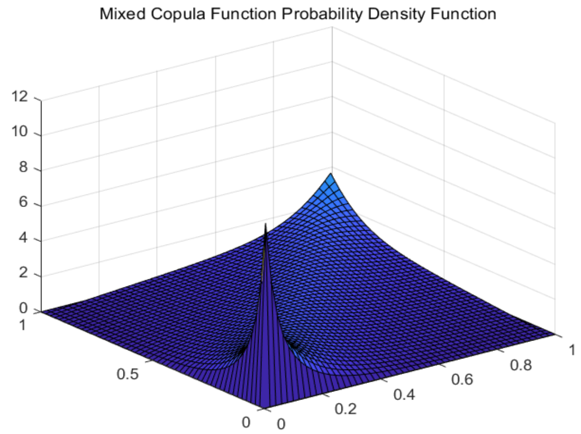

FIGURE 3: Mixed Copula function fitting results

The residual sum of squares between the fitting result of the mixed Copula function and the original data is 0.0163, which is better than the fitting result of any single Copula function, indicating that the mixed Copula function combines the characteristics of different Copula functions well.

4.5. Determination of Corn Insurance Rate Based on MCSO Model.

From 3.4 and 3.5 above, it can be seen that the objective function of this paper can be determined as the lowest variance of corn sales income after farmers apply for insurance, and the decision variables are selected as insurance amount and insurance rate. Finally, according to 3.5, the constraints of this example are set. Risk constraint and profit constraint do not need to be changed, and can be solved by substituting the obtained joint distribution function of corn yield and price. In the selection of the constant MA in premium constraint, this paper selects the average value of the corn income insurance premium in the 18 regions in Henan Province's Corn Income Insurance Pricing and Product Design Research [30] as the limit, that is, the premium should not exceed 25.2261 per mu of land. Yuan. Based on this, the MCSO model is established as follows.

The decision variables of the model are the insurance amount PI and the insurance rate PR.

Model objective function:

$$min\ LS = \iint (xy+C)^2 f(x,y)dxdy - (\iint (xy+C) f(x,y)dxdy)^2.$$

Model constraints:

$$S.T\begin{cases} LS < FS \\ \iint (P-C) * f(x,y)dxdy \geq 0. \\ PI \times PR < 25.2261 \end{cases}$$

After solving the MCSO model, the final insurance amount is 1124.16 yuan per mu, the insurance rate is 2.24%, and the risk is reduced by about 41.45% compared with the pre-insurance.

5. Summary and Outlook

Agriculture is the top priority of China's development. In recent years, the agricultural insurance premium subsidy policy and the rising agricultural insurance premium income which have been continuously issued by the central government, indicates that agricultural insurance is in a period of rapid development. The MCSO model proposed in this paper is more applicable to the rate determination of general income insurance products. When encountering a more complicated rate determination situation, such as the situation that requires the insurance company's income to be higher than a certain value, which cannot be solved by the conventional Monte Carlo stochastic simulation method, the MCSO model in this paper can be considered to solve the rate.

5.1. Advantages of MCSO model over traditional Monte Carlo method.

In the process of rate determination, the mainstream method is generally to use the fitted Copula function to calculate the average income of crops as the insured amount, and to calculate the insurance rate by calculating the expected loss of crops. The advantage of this method is that it can solve most agricultural insurance rate problems, but its disadvantage is that it is less flexible. In the process of determining the rate, only the expected loss of crops is considered, and the insurance premiums paid by farmers and the income of insurance companies are not considered, which is not in line with the actual situation. The MCSO model can control this kind of actual situation very well. However, when using the MCSO model in this paper, there are several problems that need to be paid attention to. Ignoring these problems will also lead to a large deviation between the results and the actual situation.

5.2. Matters needing attention when using the MCSO model.

When using the MCSO model, there are several situations that need to be paid attention to. Once ignored, the results may deviate greatly from the actual situation or cannot be solved.

(i) The maturity cycle of crops is not one crop per year or more than one year

The influence of different types of crops is mainly due to the different maturity cycles of crops. The unit yield and price selected in this paper are the values of crops within one year, that is, the unit yield and price are the average yield and price within a year. At this time, it is still applicable to crops with one crop per year or more than one year. However, this selection is not applicable to crops such as three crops in two years, and the selection method needs to be changed, which can be replaced by the average yield and price of the crops within two years.

(ii) Mixed Copula and single Copula results are the same or similar

In the fitting of mixed Copula functions, there may be a case where the weight of a Copula function is 1 when fitting the yield or price of some crops. This happens because the mixed copula fits better than the single copula in most cases, but not all the mixed copulas outperform the single copula. When this happens, consider adding a Copula function or select a single Copula function for fitting.

(iii) The integral in the stochastic optimization model is difficult to solve by definition

When calculating the benefits and risks of crops, there will be situations where the integration is very complex and difficult or impossible to solve. In this case, the integral definition can be used, and the integral interval is divided into enough equal-length cells to sum up to approximate the actual integral value. Methods.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was funded by National College Students Innovation and Entrepreneurship Training Program (202010058004) and Southern Xinjiang Research Center of Agricultural Information (zx2015003).

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