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Study on the relationship between deformation and load of triangle embedment in wood

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Abstract

In the wooden structure connections, especially at the mortise-tenon joints of the oriental traditional timber structure, the triangular embedment of wood is the main factor that dominates its rotational performance, so it also plays an important role in the seismic resistance of timber structure. The study of the relationship between deformation and load of wood in the process of triangular embedment is the premise of analyzing the rotational performance of wood structure joints under lateral load. In this paper, by considering the orthotropy of wood, the deformation of wood surface under triangular preloading is calculated by using Pasternak model, and the relationship between deformation and load is obtained. This workshop is helpful to the study of the mechanical behavior of traditional timber mortise-tenon joints.

Keywords

Embedment of wood, traditional timber structure, deformation, Pasternak model.

1. Introduction

In the long river of human history, wood is a very important building material. Before the emergence of steel, cement and other modern building materials, it used to be the most widely used material for people to use in housing construction. Even now, there are also many timber structure houses in construction and use, and there are many heritage timber structure buildings with historical and cultural value that need us to protect and preserve. As an organic and green building material, wood plays an irreplaceable role in reducing carbon emissions of Greenhouse Effect. But at the same time, the physical and mechanical properties of wood will degenerate after long-term use, and it will be corroded by fungi and eaten by worms in humid environment. In addition, what is more important in the mechanical properties of wood, it is a kind of anisotropic material, and there are significant differences in radial, transverse and tangential mechanical properties of wood. In the timber structure, especially in the traditional timber structure, the wood is often compressed in the transverse direction, which is the most common in the connection of the timber structure, and the tenon is usually inserted into the joint of the mortise. This kind of joint will rotate in the plane when the timber structure is subjected to lateral load, such as earthquake. Because the compressive strength of the wood parallel to grain is much greater than that of the wood perpendicular to grain, the wood parallel to grain of the mortise is pressed into the tenon to form a triangular embedded pressure in the joint. This unique mechanical behavior of wood belongs to the category of large strain problem of materials, which is different from the small strain problem of steel, concrete and other materials. The rotational stiffness of the timber structure joint is obtained by the triangular compression of the tenon and the mortise, so as to provides the lateral stiffness to the whole structure to resist the seismic damages. Therefore, the elastic-plastic performance of wood triangle embedment is very important in the study of seismic performance of traditional wood

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structures, and it is also the basis for evaluating the seismic vulnerability of traditional wood structures.

In order to obtain the load-deformation relationship of wood triangle embedment, the first and the most important is to find out the deformation law of wood surface in the process of triangle compression, and then express it with a suitable curve, so as to get the deformation function. On this basis, the deformation volume of wood under triangular embedment can be obtained by integral, so that the corresponding relationship between deformation and load can be established through the constitutive relation of wood. It can be seen that the function expression of wood surface deformation in triangle compression is the most important content, and many scholars have carried out in-depth research on this problem and achieved a lot of results. At present, Pasternak model is the most suitable method, and it is also a calculation method verified by experiments. Therefore, in this paper, the Pasternak model is applied to the triangular compression of wood at the joints of traditional timber structures, and finally the theoretical relationship between deformation and load of wood triangle embedment is deduced.

2. Triangle embedment in wood

2.1. Anisotropy of wood

Wood is a typical anisotropic material, as shown in figure 1. The compressive strength, tensile strength and shear strength of wood are very different in the direction of parallel to grain and perpendicular to grain. Taking pine and fir as examples, the wood mechanical properties obtained by experiments are shown in Table 1. It can be seen that the compressive strength and elastic modulus of wood parallel to grain are larger than that of wood perpendicular to grain. In some literatures, it is usually assumed that the elastic modulus of wood parallel to grain is 20 times that of wood perpendicular to grain. For this reason, when the wood parallel to grain is extruded with the wood perpendicular to grain, the deformation almost all comes from the wood perpendicular to grain, as shown in figure 2.

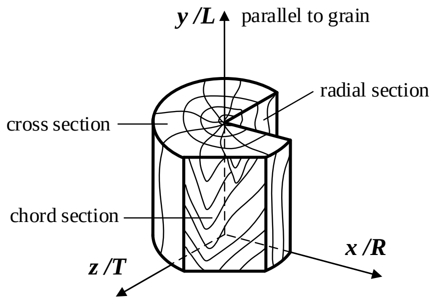


Figure 1 Anisotropy of wood

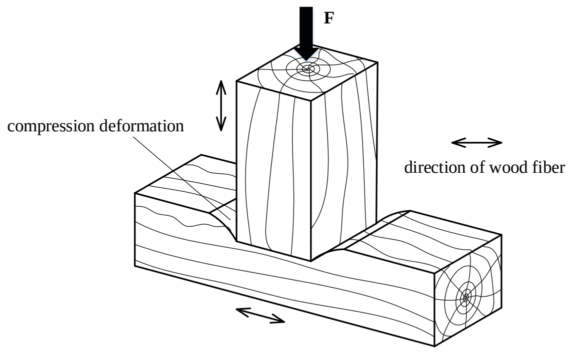


Figure 2 Compression deformation of wood perpendicular to grain

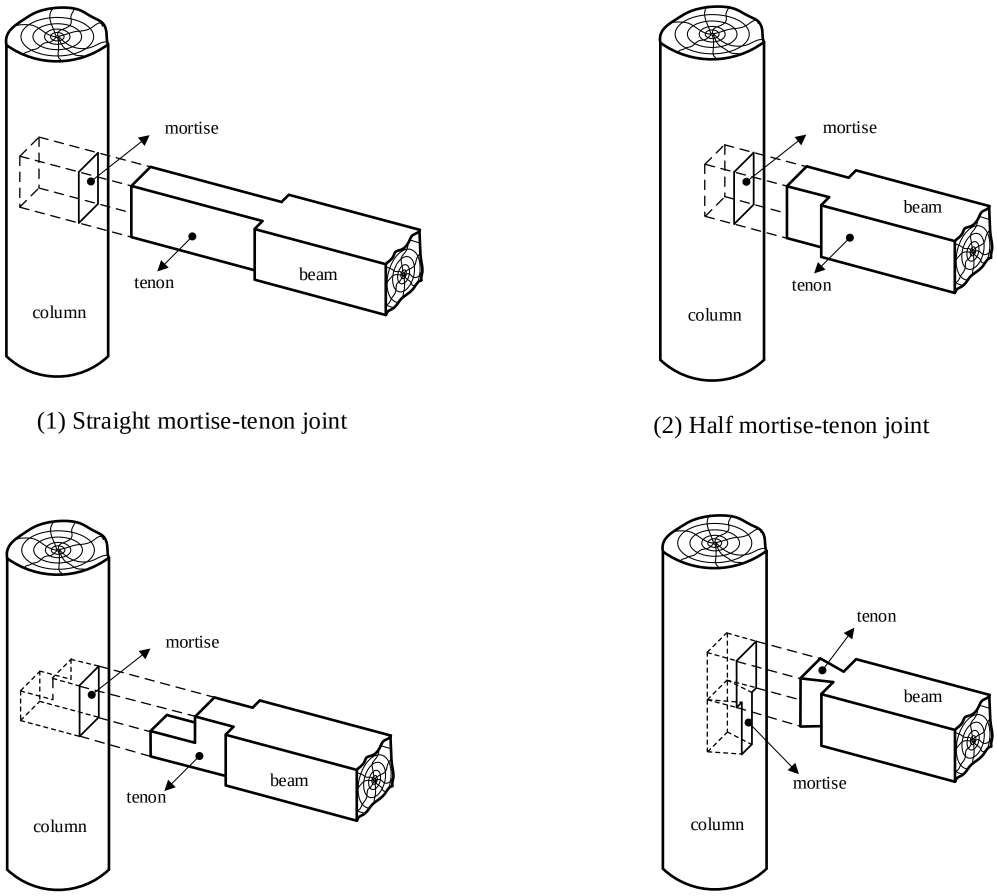
Table 1: Mechanical properties of pine and fir wood

Mechanical properties		Wood species: pine	fir
Compressive strength of wood parallel to grain $/f_{c,L}$ (MPa)		43.6	35.1
Compressive strength of wood perpendicular to grain (MPa)	Radial direction $/f_{c,R}$	5.0	3.7
	Chord direction $/f_{c,T}$	7.5	3.7
Bending strength of wood $/f_m$ (MPa)		62.7	56.8

Elastic modulus of wood parallel to grain /$E_{c,L}$ (MPa)		11120	7410
Elastic modulus of wood perpendicular to grain (MPa)	Radial direction /$E_{c,R}$	1158	830
	Chord direction /$E_{c,T}$	640	510

2.2. Triangular embedment of wooden joints

Chinese traditional timber structure is the representative of oriental architecture, and its structure is very unique, which is very different from the western architectural system, of which the most characteristic is the connection form between wood components. The wooden joints are assembled by inserting the tenon which protruding from the end of the beam into the mortise notch on a wooden post, which is the so-called mortise and tenon joints. From the perspective of archaeological research, this form of joint first appeared in the Neolithic Age, which was more than 7000 years ago. After that, it gradually developed and spread to all parts of the world dominated by East Asia, such as Japan, South Korea, Singapore, Vietnam and other countries. Common timber structure mortise and tenon joints are straight mortise-tenon joint, half mortise-tenon joint, step-penetrated mortise-tenon joint, dovetail mortise-tenon joint and so on, as shown in figure 3.



(1) Straight mortise-tenon joint

(2) Half mortise-tenon joint

(3) Step-penetrated mortise-tenon joint

(4) Dovetail mortise-tenon joint

Figure 3 Common mortise-tenon joints

Under the action of earthquake, the whole structure of the traditional timber structure will have lateral deformation, and the mortise-tenon joints will rotate accordingly, and the mortise and tenon will be subjected to triangular compression. As shown in figure 4, the wood parallel to

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grain of the mortise is embedded in the wood perpendicular to grain of the tenon in Japanese Nuki joint during rotation, resulting in a significant triangular embedment behavior.

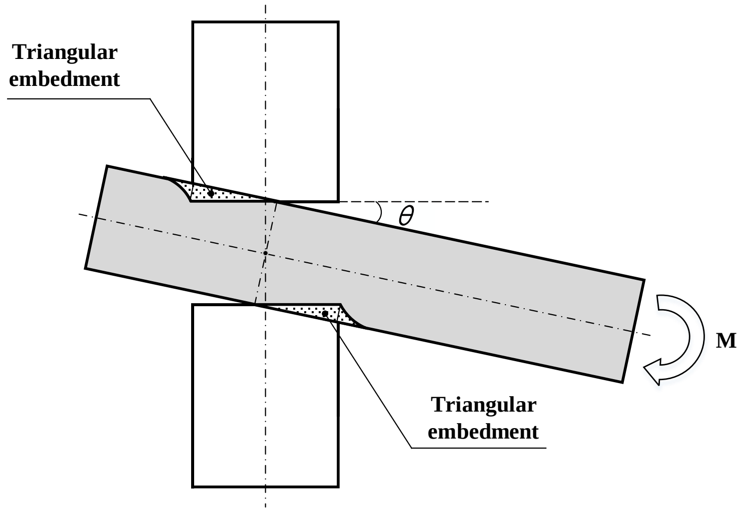

Figure 4 Triangular embedment of Nuki joint in Japan

3. Pasternak model in wood

3.1. Pasternak model

There are many types of mechanical models of continuum, among which Winkler model is the simplest and easiest to deal with mathematically, but there are still some problems when Winkler model is directly used in wood embedded pressure, mainly because it can not reflect the continuity of adjacent spring displacement. Therefore, Pasternak made necessary improvements in 1954 and put forward the Pasternak model (hereinafter referred to as the PM model). Adding the shear layer to the Winkler model can better express the surface displacement distribution of the continuum under local pressure load. The PM model is a two-parameter model. The first parameter has the same meaning as the spring stiffness k in the Winkler model and is similar to the base reaction coefficient in geotechnical engineering. The second parameter reflects the continuity between the adjacent springs and is expressed by the vertical shear stiffness G, but the shear stiffness here is different from the shear model of materials.

3.2. Pasternak model in wood

The Pasternak model can represent the continuous surface displacement under embedded pressure, which is very effective for the embedded pressure at the end of wood beam. The governing equation is expressed as a second order differential equation (1).

$$p = ky - G\frac{d^2 y}{dx^2} \tag{1}$$

Where y is the deformation, P is the vertical load, k is the foundation reaction coefficient, and G is the shear stiffness per unit width of the shear beam, which only produces the shear deformation or shear stiffness per unit width of the soil layer in the vertical direction.

The solution of vertical line load P is as follows:

$$y = \frac{P\gamma}{2k} e^{-\gamma|x|} \tag{2}$$

Where γ is the eigenvalue of the model, and

$$\gamma = \sqrt{k / G} \tag{3}$$

Pasternak model in wood is shown in figure 5.

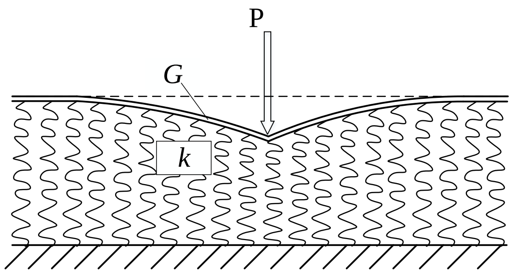

Figure 5 Pasternak model in wood

4. Relationship between deformation and load of triangle embedment in wood

4.1. Basic hypothesis

In order to deduce the relationship between deformation and load during wood triangle compression, some factors with little influence are simplified, and the basic assumptions are as follows:

(1) It is found that the stress-strain curve of wood under compressive force perpendicular to grain generally goes through three stages: linear, softening yield and strain hardening, as show in figure 6. Considering the deformation of wood joints in the actual structure, the wood transverse grain embedded pressure will not reach the strain hardening stage. In order to simplify the constitutive relation of wood, it is assumed that the stress-strain curve of wood cross-grain compression consists of two straight lines, as shown in figure 6.

(2) The deformation properties of wood in all directions are different from each other. In general, the compression elastic modulus of wood parallel to grain is 10-15 times of that perpendicular to grain, so when local embedment occurs, relative to the compression of perpendicular to grain on the contact surface of tenon, the deformation parallel to grain on the contact surface of mortise is very small and can be ignored. In other words, it is considered that the deformation occurs only in the embedded part of the upper and lower side of the tenon.

(3) In the process of wood deformation, the vertical load P is always parallel to the z-axis.

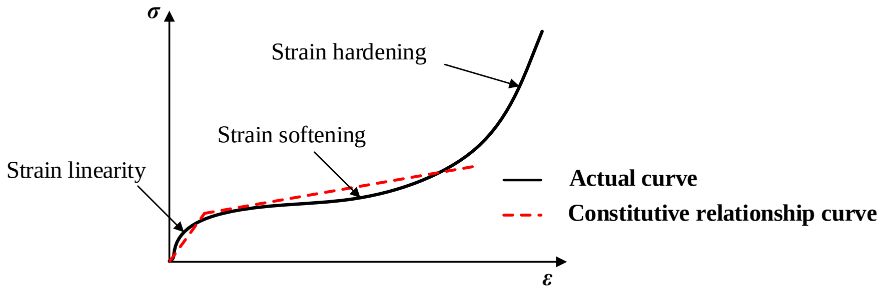

Figure 6 Constitutive relation of wood perpendicular to the grain

4.2. Relationship between deformation and load of triangle embedment in wood

The schematic diagram of the triangular embedment of the full section of wood is shown in figure 7. According to the basic assumptions and the Pasternak model in the wood, the deformation of the wood surface is divided into two parts. Part I represents the deformation of the loading region, the deformation curve is a straight line, and the functional relationship is formula (4). Part II represents the deformation of the unloaded region, and its deformation curve is a curve, and the functional relationship is as follows (5).

$$y=\theta x \tag{4}$$

$$y=he^{-\alpha x} \tag{5}$$

According to the reference [5], when the wood is elastic, the value of α can be deduced according to the following formula:

$$\alpha=\frac{3}{2z_1}=\frac{1.5}{H} \tag{6}$$

Here z_1 represents the thickness of the wood perpendicular to grain in the compression direction, which is the height of wood H in figure 7.

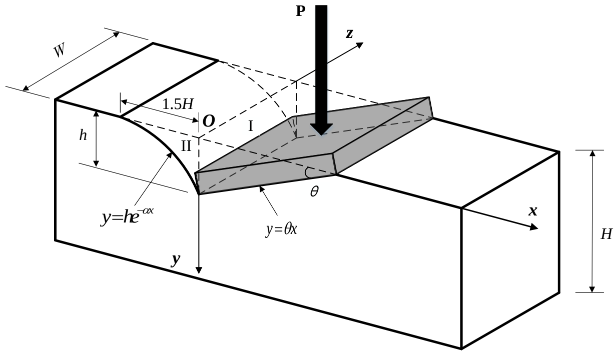

Figure 7 Triangular embedment of wood

According to the geometric relationship in figure 7, it can be obtained:

$$h=L\sin\theta \tag{7}$$

From this, the deformation volumes V_I and V_{II} of the embedment zone in the elastic stage can be calculated as follows:

$$V_I=\frac{1}{2}WLh\cos\theta=\frac{1}{2}WL^2\sin\theta\cos\theta\approx 0.5WL^2\theta \tag{8}$$

$$V_{II}=W\int_0^{1.5H}he^{-\frac{1.5}{H}x}dx=0.6LWH\sin\theta\approx 0.6LWH\theta \tag{9}$$

Furthermore, the resistance force P' caused by the compression deformation of wood perpendicular to grain is obtained as follows:

$$P'=WE_{c,R}\int_{x_2}^{x_1}\frac{y(x)}{H}dx=\frac{E_{c,R}}{H}V=\frac{E_{c,R}}{H}(V_I+V_{II}) \tag{10}$$

By substituting the formula (8) and (9) into the formula (10), and consider that the resistance force P' caused by the compression deformation is numerically equal to the applied load P, but in the opposite direction: it can be obtained:

$$P=\frac{E_{c,R}}{H}(V_I+V_{II})=E_{c,R}WL(0.5L/H+0.6)\theta \tag{11}$$

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