Parameter Estimation of Frequency Hopping Signal Based on Modulated Wideband Converter

Shiru He, Zhi Li * and Jian Li
School of Sichuan, Sichuan University, Chengdu 610000, China
Corresponding author

Abstract

To resolve the problems of high hardware implementation cost and low reconstruction accuracy of existing traditional greedy algorithms. This paper proposes a Bayesian orthogonal matching pursuit (BOMP) algorithm based on modulated wideband converter (MWC) to estimate the parameters of frequency hopping (FH) signals. In the frequency domain, the algorithm estimates its parameters through the sparsity of the frequency hopping signal. The FH signal is sampled by the MWC framework and reconstructed by the BOMP reconstruction algorithm. The time-frequency ridge method is then adopted to estimate the hopping duration, hopping time, and carrier frequency of the time-frequency diagram. The simulation results show that in a low signal-to-noise ratio (SNR) environment, the anti-noise performance is improved by 6dB compared with traditional methods. This method improves the accuracy of parameter estimation of frequency hopping signal and dramatically reduces the hardware pressure.

Keywords
MWC, BOMP algorithm, frequency hopping signal, parameter estimation.

1. Introduction

As a spread spectrum signal commonly used in real life, frequency hopping communication [1] has the advantages of strong confidentiality, strong anti-interference ability, low interception probability and strong networking ability, and is widely used in the military field. However, since the traditional sampling technology needs to meet the Nyquist sampling rule, its high sampling rate, large amount of data processing and relatively large power consumption limit the development of low-power devices. The compressed sensing (CS) technology [2] proposed in 2006 breaks through the Nyquist sampling theorem in the digital domain, compresses the signal, relieves the pressure of back-end storage and transmission, and also provides a new idea for sparse signal sampling.

For the current research on frequency hopping signal parameter estimation [3-5], the existing methods usually require higher hardware conditions and cannot adapt to lower signal-to-noise ratios. In order to solve the hardware pressure problem in frequency hopping signal communication, some scholars [6, 7] introduced MWC into the frequency hopping signal system. The experimental results show that MWC can greatly reduce the hardware pressure in frequency hopping communication. In the literature [8], the author uses the sample data correlation after MWC sampling to estimate the carrier frequency parameters of the frequency hopping signal. This method does not need to reconstruct the sampled signal, which reduces the amount of calculation, but the anti-noise performance poor. In the literature [9], the author uses the multi-measurement sparse Bayesian method to estimate the parameters of the frequency hopping signal, and introduces the Bayesian model, which has achieved good results, but the method has the assumption that the distribution is independent, and the real life It is difficult for these predictions to be completely independent.
On the basis of the above research, this paper proposes a new method that combines the BOMP reconstruction algorithm with MWC. This method uses the MWC model for sampling to reduce the pressure of hardware in practical applications, and at the same time uses the BOMP algorithm to remove the reconstruction process. The influence of noise on the support set facilitates parameter estimation under low signal-to-noise ratio conditions.

2. Parameter Estimation of FH Signal Based on MWC-BOMP Algorithm

2.1. MWC system description

MWC is a multi-channel system. Each channel has the same structure and consists of a low-pass filter, a mixer, and a low-speed sampler. The system structure is shown in Figure 1. Among them, \( x(t) \) is a multi-band signal; \( m \) is the number of channels; \( p(t) \) is a mixing signal with a period of \( T_p \); \( h(f) \) is an ideal low-pass filter with a bandwidth of \( 1/2T_{NYQ} \); \( T_s \) represents the ADC sampling interval; \( y_i[n] \) represents the sequence acquired by sampling on the \( i \)-th channel, where \( i = 1, 2, \ldots, m \).

\[
\text{Figure 1. MWC system structure diagram}
\]

2.2. BOMP Recovery Algorithm

The BOMP algorithm is based on the OMP algorithm and combined with the Bayesian test model. Because noise generally exists in the real environment, the equation \( v = Au + e \) is expressed as

\[
v = Au + e
\]  

Among them, it is assumed that the noise vector \( e \) obeys the Gaussian distribution \( e \sim N(0, \sigma^2_e I) \) (\( I \) is the identity matrix), and the sparse signal \( u \) adopts the Bernoulli Gaussian model. If the probability of the \( i \)-th element \( u_i \) taking a zero value is \( p \), The corresponding probability of taking a non-zero value is \( 1 - p \), and the \( i \)-th component of \( u \) is \( u_i = q_i r_i \), and \( r_i \) obeys the Gaussian distribution \( r_i \sim N(0, \sigma^2_r) \), and when \( u_i \neq 0, q_i = 1 \), otherwise \( q_i = 0 \), then \( u \) can be expressed as \( u = Q r \), where \( Q \) is a diagonal matrix with the elements of vector \( q \) as diagonal elements.

This algorithm aims to reduce the faulty redundant components of initial sparsity by constructing a Bayesian test model. If the parameter \( p \) is known, let the rough sparsity of the sparse signal be \( K \) (\( K \) is only a redundant initial value set based on experience, not strictly equal to the signal sparsity). Note that \( K = \|q_0\| \), \( (q_0 \) represents the number of elements that not equal to zero in vector \( q \)), the probability of vector \( q \) is

\[
p(q|p) = (1 - p)^{\|q\|_0} p^{L - \|q\|_0}
\]
If \( w \) is used to represent a vector composed of all non-zero elements of the sparse signal \( u \), then \( \| w \|_2^2 = \| Qr \|_2^2 \). The probability density function of the vector \( w \) under the condition that \( \sigma_r^2 \) is known is

\[
p(w \mid \sigma_r^2) = \left( \frac{1}{2\pi\sigma_r^2} \right)^\frac{K}{2} e^{- \frac{\| w \|_2^2}{2\sigma_r^2}} = \left( \frac{1}{2\pi\sigma_r^2} \right)^\frac{\| q \|_0}{2} e^{- \frac{\| Qr \|_2^2}{2\sigma_r^2}} \tag{3}
\]

Since the noise vector \( e \) obeys the Gaussian distribution \( N(0, \sigma_e^2I) \), the conditional probability of the observed signal \( v \) when \( \sigma_e^2, Q, r \) is known is

\[
p(v \mid \sigma_e^2, Q, r) = \left( \frac{1}{2\pi\sigma_e^2} \right)^\frac{m}{2} \exp\left( - \frac{\| v - Au \|_2^2}{2\sigma_e^2} \right) = \left( \frac{1}{2\pi\sigma_e^2} \right)^\frac{m}{2} \exp\left( - \frac{\| v - AQR \|_2^2}{2\sigma_e^2} \right) \tag{4}
\]

In the Bayesian hypothesis testing model, the parameters \( p, \sigma_r^2, \sigma_e^2 \) need to be estimated with the maximum posterior probability. Therefore, the first condition is to obtain the largest posterior probability estimate of each parameter. In other words, to calculate the derivation of the parameters \( p, \sigma_r^2, \sigma_e^2 \) from Eq. (2) to Eq. (4) and set it to 0, the maximum posterior probability estimate obtained value is

\[
\hat{p} = \frac{L - \| q \|_0}{L} \tag{5}
\]

\[
\hat{\sigma_r^2} = \frac{\| w \|_2^2}{\| q \|_0^2} = \frac{\| Qr \|_2^2}{\| q \|_0^2} \tag{6}
\]

\[
\hat{\sigma_e^2} = \frac{\| v - AQR \|_2^2}{M} \tag{7}
\]

In order to eliminate the redundant support set atoms contained in the candidate set output by the orthogonal matching pursuit algorithm, the likelihood ratio function is constructed according to the Bayesian hypothesis testing model. For the detailed derivation process, refer to [10], and the likelihood ratio test formula is:

\[
(z_j - m_j)^2 > H_1 \quad \text{if} \quad H_1 \quad \text{else} \quad H_0 \quad \text{Th}_j \tag{8}
\]

where

\[
\text{Th}_j = \frac{2\sigma_{r,j}^2}{b_{jj}^2\sigma_r^2} \left( \sigma_{r,j}^2 + b_{jj}^2\sigma_r^2 \right) \ln \left( \frac{p}{1-p} \sqrt{\frac{\sigma_{r,j}^2 + b_{jj}^2\sigma_r^2}{\sigma_{r,j}^2}} \right) \tag{9}
\]

According to the Eq. (8) criterion can be seen that the event \( H_1 \) is established, the value of the original signal \( u_j \) corresponding to the subscript \( j \) is non-zero, so the value corresponding to the subscript \( j \) is retained. Otherwise, the \( H_0 \) event is established, and the subscript \( j \) corresponds to the value of the original signal \( u_j \). If the value is zero, delete the support set corresponding to the subscript \( j \). Finally, the signal is reconstructed by the support set with the redundant support set atoms removed. The algorithm flow is shown in Table 1 below.

**Table 1. BOMP algorithm steps**

**Algorithm1: the method of BOMP**

Input: observation vector \( V \), perception matrix \( A \), coarse sparsity \( K \);
Initialization: initial residual value $R_0 = V$, support set $S_0 = \emptyset$, iteration number $t = 1$.

1. Use the OMPMMV algorithm to coarsely reconstruct the signal to obtain the support set $S_t$ and the coarsely reconstructed signal $\hat{U}$;

   1.1. Select the $k$-th column of matrix $A$, which satisfies $k = \text{argmax}_k \|X_k\|_q$, where $X_k = R_{t-1}^T A_k$;

   1.2. Update the support set $S_i = [S_{i-1}, k]$, save the recovery vector set $A_{S_i} = [A_{S_{i-1}}, A_k]$;

   1.3. Estimate $\hat{U}, \hat{U}_i = A_{S_i}^T V$;

   1.4. Update the residual $R_i = V - A_{S_i} \hat{U}_i, i = i + 1$;

2. Taking the support set $S_t$ as the candidate set, let its dimension be the estimated value of $\|q\|_0$, the rough reconstructed signal $\hat{U}$ is the estimated value of $Qr$, and calculate the threshold $Th_j$. Then filter the elements in $S_t$, and all elements larger than the threshold are retained in the set $F$ as the final support set;

3. According to the final support set $F$, use the least square method to gain the estimated final value $\hat{U}$ of the sparse signal.

Output: support set $F$, reconstruction signal $\hat{U}$;

2.3. Parameter estimation method based on time-frequency ridge method

For the FH signal of a single network station, the time-frequency ridge method is the main method for blind parameter estimation [11-13] because of its simple operation. Since the time-frequency ridge diagram usually contains various parameter information of the frequency hopping signal, the time-frequency ridge diagram is usually obtained by the time-frequency analysis of the frequency hopping signal for subsequent parameter estimation. A time-frequency ridge graph is a graph of the maximum frequency of a frequency-hopping signal over time. Usually, in order to weaken or eliminate the interference of noise, it is also necessary to perform first-order difference processing on the time-frequency ridge graph. The algorithm flow is shown in Table 2 below.

Table 2. the time-frequency ridge algorithm steps

<table>
<thead>
<tr>
<th>Algorithm 2: the time-frequency ridge algorithm</th>
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<tbody>
<tr>
<td>1. The reconstructed frequency hopping signal is processed by the time-frequency analysis method, and the rough time-frequency diagram and time-frequency matrix are obtained;</td>
</tr>
<tr>
<td>2. In the time-frequency matrix, extract the frequency corresponding to the value with the largest amplitude at each moment, and form a time-frequency ridge line with time;</td>
</tr>
<tr>
<td>3. Calculate the first-order difference of the time-frequency ridge line to obtain the frequency hopping timing diagram;</td>
</tr>
</tbody>
</table>
4. Frequency hopping time The time of the first frequency hopping in the diagram is the jumping time, and the average value of the duration of each carrier frequency is the frequency hopping period.

3. Experiments

In order to verify the performance of the BOMP frequency hopping signal parameter estimation algorithm based on MWC proposed in this paper, a single network frequency hopping signal is simulated and generated, and the experiments are compared with the three algorithms of sampling in Nyquist, MWC-OMP sampling and reconstruction, and MWC-BOMP sampling and reconstruction. The parameter estimation error of frequency hopping time, frequency hopping period, and frequency hopping frequency is as follows.

Parameter setting: FH signal hopping period: \( T_h = 7\mu s \), modulation mode: quadrature phase-shift keying (QPSK), number of frequency bands: \( N = 6 \), frequency variation range: \( 0\sim250MHz \), Nyquist frequency: \( f_{nyq} = 2GHz \), 4 hops are selected as the experimental target signal in the observation range, the frequency is set to \( \{50,180,220,100\}MHz \), the observation time is \( 28\mu s \), the signal-to-noise ratio \( SNR = 10dB \). The parameters of the MWC system are set as follows: the number of channels \( m = 30 \), the pseudo-random sequence length is \( L = 109 \), each channel's sampling frequency is \( f_s = f_p = f_{nyq}/L \), here the short-time Fourier transform (STFT) is used time-frequency analysis method, and set the window function to a Hamming window with a length of 255.

Evaluate its performance by estimating the average absolute error of the FH period:

\[
ER = \frac{1}{RT} \sum_{i=1}^{N} |\hat{T}_i - T|
\]  

Among them, \( \hat{T}_i \) is the FH period obtained in each test. In theory, \( T \) is the value of the frequency hopping period. \( N \) is the number of experiments. The SNR range is from \(-10dB\) to \( 20dB \), and the number of experiments under each SNR is \( 200 \) times. Figure.2 illustrates the estimated average absolute error of the FH period of OMP and BOMP methods under different SNR environments.

![Figure 2. Frequency hopping period estimation error](image)

Figure 2 shows that with the gradual increase of the SNR, the period estimation errors of the three methods are all decreasing. And, the effect of the BOMP recovery algorithm is obviously superior to the OMP recovery algorithm. Assuming that the parameter value of the FH period could be more accurately estimated when the estimation error is less than or equal to 0.1, the
BOMP algorithm can more accurately estimate the parameter value of the FH period when the SNR is about 4dB. The OMP algorithm can only estimate accurately while the SNR is 10dB. The average absolute error of hop time estimation is defined as:

$$ER = \frac{1}{RT_1} \sum_{i=1}^{N} |\hat{t}_i - T_1|$$  \hspace{1cm} (11)

Among them, $\hat{t}_i$ is the estimated value of the take-off time of each test. In theory, $T_1$ is the value of the take-off time. $N$ is the number of experiments. Figure 3 shows that the average absolute error of FH signal hoping time of the OMP algorithm and the BOMP algorithm under the environment of SNR of $-10dB$ to $20dB$.

![Figure 3. Hop time estimation error](image)

Figure 3 shows that with the increase of SNR, the errors of the three methods decrease rapidly. When SNR is 4dB and 11dB, respectively, the estimation error value of the BOMP method and OMP method is less than 0.1 for the first time, so that the BOMP method can estimate the hop time more accurately.

The average variance of the normalized carrier frequency estimate is defined as:

$$\sigma = \frac{1}{Rk} \sum_{l=0}^{k} \sum_{i=1}^{N} \left( \frac{\hat{f}_{il}}{f_i} - 1 \right)^2$$  \hspace{1cm} (12)

In the observation time, $k$ is the overall number of hops of FH signal, $\hat{f}_{il}$ is the estimated value of the normalized FH signal carrier frequency of the $i$-th hop in the $l$-th time, $f_i$ is the normalized carrier frequency of the $i$-th hop, and $N$ is the number of experiments. Figure 4 shows under different SNR environments, a comparison of the normalized carrier frequency's error of average absolute by different methods.

Figure 4 shows that when the SNR of the BOMP algorithm is $-1dB$, the normalized carrier frequency estimation error is less than 0.1, while the error of the OMP algorithm is less than 0.1 for the first time when the SNR is 3dB. The BOMP sampling recovery is significantly better than OMP sampling recovery and can estimate parameters more accurately under low SNR.
4. Conclusion

This study proposes a BOMP algorithm based on MWC, which uses MWC to sample the signal and reduces hardware transmission pressure. Besides, a Bayesian hypothesis testing model for identifying the support set of FH signals in a noisy environment is established. The redundant support set in the output support set of the OMP algorithm is eliminated according to the model. Finally, according to the support set, to reconstruct and estimate the parameters of the FH signal. The simulation results show that the performance of this algorithm is significantly better than existing algorithms in the estimation of FH signal parameters. This method is more suitable for low SNR environments.

References


