

# Research on Hydropower Resource Allocation Based on Multi-Objective Programming and Genetic Algorithm

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## Abstract

Due to climate change, historical agreements and other factors, the amount of water available from dams and reservoirs is decreasing in many areas. This has led to increased competition for hydropower resources in the region and in some nearby states. In this paper, we focus on building a programming model to solve the hydropower resource allocation problem. We define the best separate allocation as the one with the shortest transportation time, the most equitable distribution, and the best satisfaction of each state. Then we establish a multi-objective programming model based on the coupling relationship between the two reservoirs, the consistent of total water, neglect of energy loss and some other constraints. Here the model is solved by genetic algorithm based on the average water levels of Lake Mead and Lake Powell in 2020, resulting in 10 Mgal/d for Lake Mead pumping and 12 Mgal/d for Lake Powell pumping; an average annual water level of up to 15 days to meet the water needs of all states; and a cumulative total of 560.24 Mgal of additional water must be provided to ensure fixed demand: finally, a sensitivity analysis is performed by varying the number of populations in the genetic algorithm to show that the model is stable.

## Keywords

Multi-objective programming, genetic algorithm, hydropower resource allocation.

## 1. Introduction

Reservoirs, as long-used water systems throughout the ages, have provided water and electricity resources for human beings.[1] In recent years, with global warming and climate anomalies, reservoirs in many areas are decreasing, and the Colorado River Basin in the United States is facing this problem. If the drought continues, the water and electricity resources in the basin will be seriously affected.[2] Therefore, it is important that we develop a suitable model to develop a reasonable water allocation plan.[3]

## 2. Water Resource Distribution Model Based on Multi-Objective Programming

The operation of dams requires consideration of multiple factors while being influenced by each state's hydropower usage.[4] To help negotiators respond to a fixed set of water supplies and demands, this paper develops a multi-objective programming model.[5]

The shortest time to transport water

Calculate the maximum time for state  $i$  to pump water from the two lakes.

$$T_i = \max\{t_{1i}, t_{2i}\} \quad (1)$$

Then minimizing the longest pumping time in all states.

$$\min \max\{T_1, T_2 \dots T_5\} \quad (2)$$

Make water distribution more equitable

Small dams are built mainly to better manage water resources, mainly to meet the demand of the population for electricity and water consumption (including agricultural, industrial and residential water) in the surrounding states. The demand may vary from state to state and it is obvious that it is necessary to allocate a relatively balanced amount of water to different states according to the demand of each state. Here we use the Gini coefficient to determine, Corrado Gini (1912) used the Gini coefficient to measure the distribution, the Gini coefficient is generally applied to measure income inequality, as shown below

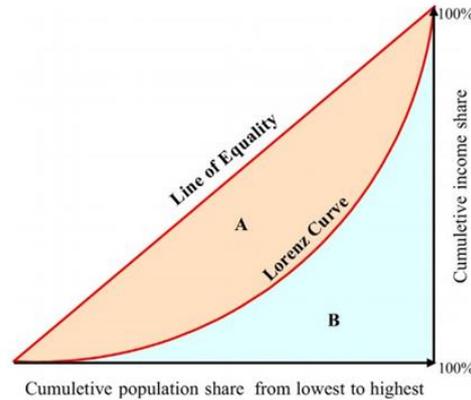


Figure 1: Lorenz curve and Gini coefficient

In the water allocation problem, the x-axis and y-axis of the graph correspond to the “cumulative economic benefits share form lowest” and “cumulative water share”, respectively, such that The Lorenz curve corresponds to the actual water allocation line. According to the definition of the Gini coefficient, the equality of water allocation is measured by the equitable distribution of the amount of water used per unit of economic benefit. Therefore, the Gini coefficient formula for water allocation is shown below.

$$Gini = \frac{1}{2m \sum_{i=1}^m \frac{IP_i}{EB_i}} \sum_{l=1}^m \sum_{k=1}^m \left| \frac{IP_l}{EB_l} - \frac{IP_k}{EB_k} \right| \tag{3}$$

Where  $m = 5$ , Gini coefficient is  $[0,1]$ , if it is close to 0, the distribution can be more equitable.

$$\min Gini \tag{4}$$

Best satisfaction index in each state

Here the satisfaction index of each state is positively correlated with the input state's electricity production and water resources. We define the satisfaction index function.

$$\max Z = \frac{w_{e_1} + w_{e_2} - \sum_{i=1}^5 a_i}{\sum_{i=1}^5 a_i} + \frac{IP_1 + IP_2 - \sum_{i=1}^5 b_i}{\sum_{i=1}^5 b_i} \tag{5}$$

The sum of the demands of each state does not exceed the sum of the reservoir inflows to each state.

$$\sum_{i=1}^5 a_i \leq \sum_{j=1}^5 \left( \sum_{i=1}^2 IP_{ij} \right) \tag{6}$$

The water storage capacity of the two lakes must not exceed their maximum carrying capacity  $C_{1max}$  and  $C_{2max}$ .

$$C_j \leq C_{jmax} \tag{7}$$

The water resources used for power generation are related to the water level in addition to the water volume, then the reservoir capacity curve equation is used to calculate the water volume to the two lakes.

$$W_1 = \frac{1}{3}(A_1 + \sqrt{A_1A_2} + A_2)P + N_{11} + N_{12} \tag{8}$$

$$W_2 = \frac{1}{3}(A_3 + \sqrt{A_3A_4} + A_4)M + N_{21} + N_{22} \tag{9}$$

It is worth mentioning that because of the coupling relationship between the two lakes, the outflow from Lake Powell is part of the inflow from Lake Mead, so  $N_2 = 0$  lakes, the outflow from Lake Powell is part of the inflow from Lake Mead, so  $N_{21} = D_1$ .  $A_1, A_2$  are the surface area of Lake Powell surrounded by each of the two adjacent contours,  $A_3, A_4$  are the surface area of Lake Mead surrounded by each of the two adjacent contours.

Disregarding the other tributaries of the lake, the conservation of the volume of the lake can be obtained as:

$$W_1 = \sum_{i=1}^5 IP_{i1} + E_1 + D_1 + C_1 \tag{10}$$

$$W_2 = \sum_{i=1}^5 IP_{i2} + E_2 + D_2 + C_2 \tag{11}$$

By ignoring the energy loss, the gravitational potential energy of the lake is completely converted into electrical energy:

$$W_{e1} = gH_1E_1 \tag{12}$$

$$W_{e2} = gH_2E_2 \tag{13}$$

Where  $g$  is the local acceleration of gravity, total water pumped is equal to the product of pumping speed and time:

$$IP_{1i} = V_1t_{1i} \tag{14}$$

$$IP_{2i} = V_2t_{2i} \tag{15}$$

In summary, we used a multi-objective optimization approach to model the decision problem of allocating Lake Powell and Lake Mead to the water resources of five states. The model is calculated as:

$$\begin{aligned} & \min \max\{T_1, T_2 \dots T_5\} \\ & \min \text{Gini} \\ & \min Z \\ & \left. \begin{aligned} & \sum_{i=1}^5 a_i \leq \sum_{i=1}^5 \sum_{j=1}^2 IP_{ij} \\ & C_j \leq C_{j\max} \\ & W_1 = \frac{1}{3}(A_1 + \sqrt{A_1A_2} + A_2)P + N_{11} + N_{12} \\ & W_2 = \frac{1}{3}(A_3 + \sqrt{A_3A_4} + A_4)P + N_{21} + N_{22} \\ & W_1 = \sum_{i=1}^5 IP_{i1} + E_1 + D_1 + C_1 \\ & W_2 = \sum_{i=1}^5 IP_{i2} + E_2 + D_2 + C_2 \\ & W_{e1} = gH_1E_1 \\ & W_{e2} = gH_2E_2 \\ & IP_{1i} = V_1t_{1i} \\ & IP_{2i} = V_2t_{2i} \\ & i \in \{1,2,3,4,5\} \\ & j \in \{1,2\} \end{aligned} \right\} \text{s. t.} \tag{17} \end{aligned}$$

### 3. Model Solving

The main tool for solving multi-objective programming by numerical methods is iterative operations. When the number of decision variables is large, the general iterative methods tend to fall into the trap of local minima and the phenomenon of “dead loops” occurs, making the iteration impossible. The genetic algorithm proposed by Professor J. Holland in 1975 overcomes this drawback and is an adaptive algorithm.[6] Genetic algorithms have the feature of generating multiple points and searching in multiple directions, which makes them ideal for solving multi-objective optimization problems with very complex search spaces for optimal solutions.[7]

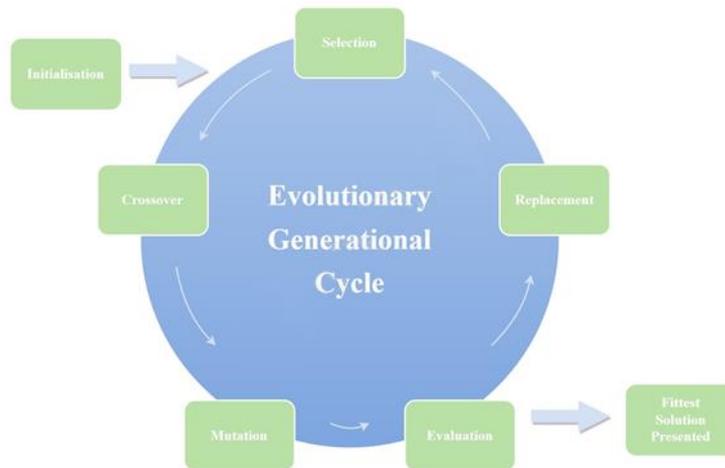


Figure 2: Genetic Algorithm flow chart

In a multi-objective optimization problem, there may be more than one optimal solution because there may be conflicts among the objectives.[8] We define the multi-objective optimal solution as follows:

Given a multi-objective optimization problem  $\min f(x), X^* \in \Omega$ , if  $\neg \exists X \in \Omega$ , such that the result satisfies the following condition: for any sub-objective function  $f_i(x)$  of  $f_i(X) \leq f_i(X^*)$ . Such a solution is also called a Pareto solution.

In the calculation process, the number of iterations is set to 500 and the population size is set to 100, where the average target value (ATV) and the trend of the best target value (BTV) are shown below:

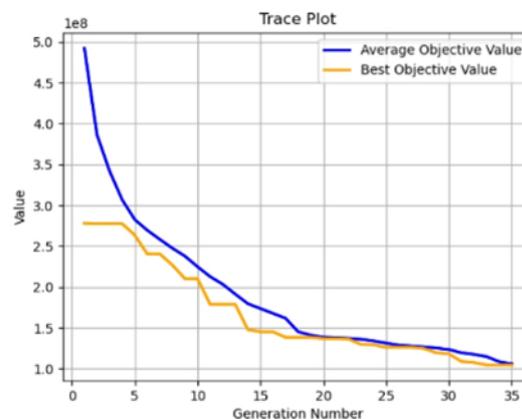


Figure 3: Trace Plot of ATV and BTV

We used the annual average water level of the two lakes in 2020. When the water level of Lake Mead  $M=1098.41$  feet and the water level of Lake Powell  $P=3596.24$  feet, Lake Mead should pump 13.16874 Mgal per day and Lake Powell should pump 11.7531 Mgal per day.

The following table shows the duration of pumping (in days) for each lake corresponding to each state:

The following table shows the duration of pumping (in days) for each lake corresponding to each state:

Table 1: Each lake corresponds to the number of days of pumping per state

	AZ	CA	WY	NM	CO
Lake Powell	8	15	2	6	2
Lake Powell	4	12	4	5	6

Considering that the water demand is fixed for each state,  $T_m = 15$  as derived in the model, an average annual water level of up to 15 days to meet the water needs of all states. Where  $T_m = \max\{T_1, T_2 \dots T_5\}$ . Since additional natural inflows such as rainfall and snowfall are not considered, the water supply to both lakes decreases over time as shown in:

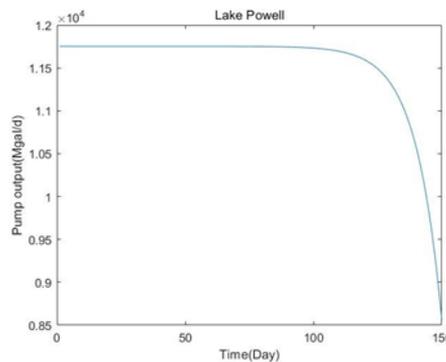


Figure 4: Daily pumping of Lake Powell

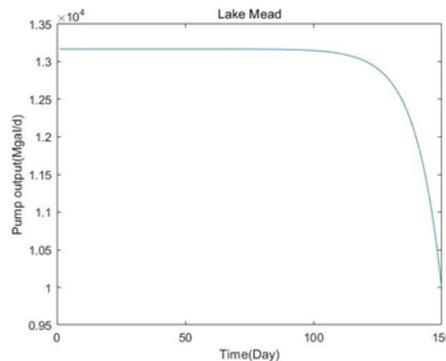


Figure 5: Daily pumping of Lake Mead

After pumping begins, Lake Mead pumping rate decreases significantly on day 100 and stops on day 155 when storage reaches its lower limit; Lake Powell pumping rate decreases significantly on day 101 and stops on day 150 when discharge reaches its lower limit. In the 155th day of the 100th day after pumping, the water resources of the two reservoirs will not be able to meet the needs of the five states, then additional water must be provided to ensure that these fixed needs are met, here additional water resources are defined as:

$$W_{addt} = \sum_{i=1}^5 a_{it} - \sum_{j=1}^2 IP_{ijt} \tag{18}$$

The variation of  $W_{addt}$  with time is shown in the following figure.

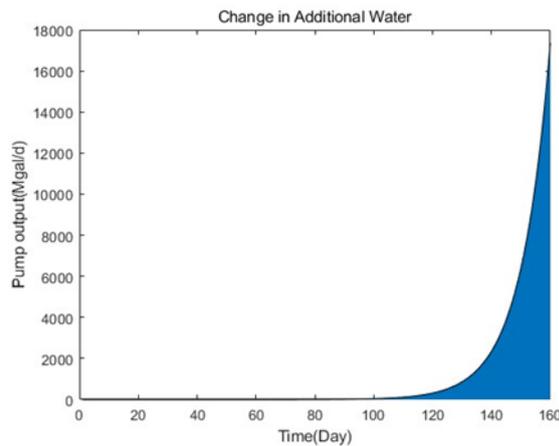


Figure 6: Daily Additional Water

The curve in Figure 6 shows that the additional daily water demand increases exponentially after 100 days, indicating that the local government must take measures to obtain additional water resources at this time. The amount of water lost during these 55 days is the area shaded in Figure 6, and measures need to be taken to obtain water from other areas to compensate for these losses, so the additional water supply is 560 Mgal.

For the solution of the multi-objective programming model, we chose the genetic algorithm. In this paper we use different population sizes to analyze the sensitivity to the genetic algorithm. Here the population size is set to increase by 25% and decrease by 25%. The trends of Average Objective Value and Best Objective Value are shown in Figure 7 and Figure 8.

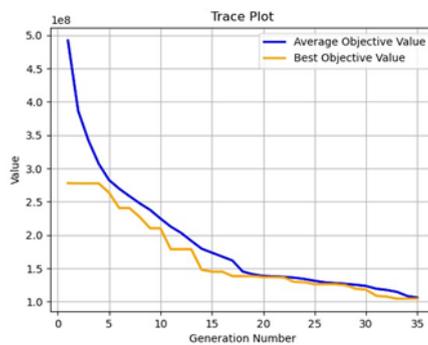


Figure 7: increase by 25%

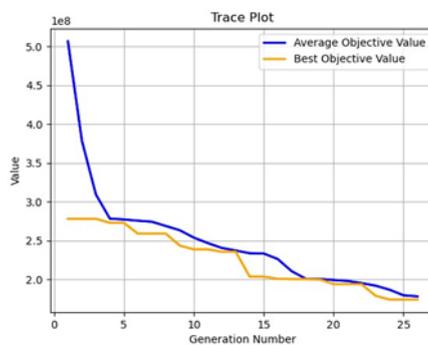


Figure 8: decrease by 25%

When the population size changes, after a certain number of iterations, the average Objective Value and Best Objective Value always match well, indicating that the algorithm is stable.

## 4. Conclusion

In this paper we developed and analyzed a multi-objective programming model that helps negotiators respond to a fixed set of water supply and demand conditions. Use this model to inform the operation of the dam: when the water level in Lake Mead is  $M$  and the water level in Lake Powell is  $P$ , how much water should be drawn from each lake to meet the stated demand. For water allocation, we use a multi-objective programming model that considers all aspects of each area. The use of bionic heuristics for solving complex models reduces the algorithmic time and space complexity.

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