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Research on trajectory tracking for nonholonomic wheeleed mobile robot

Chunchen Liao, Jun Zhao

Sichuan University of Science & Engineering, Sichuan 644000, China

Abstract

This article aim at the nonholonomic constrained robot whose geometric centroid does not coincide with the center point of the driving wheel shaft ,studied its trajectory tracking control problem based on the kinematics model of the wheeled mobile robot. In this paper, a new Lyapunov function is proposed, and a motion model controller is designed by using the function. The controller can guarantee the global uniform asymptotic stability of the trajectory tracking errors and the errors converge to zero. Finally, steady and time-varying trajectories are used as reference trajectories for tracking simulation, and the effectiveness of the control algorithm is verified.

Keywords

Nonholonomic; trajectory tracking; Lyapunov.

1. Introduction

Wheeled robot has been applied in various fields in recent years because of its lightness, flexibility and wide application scenarios[1]. Trajectory tracking control is an important research point in robot application and various scholars have studied this issue. Literature [2-4] proposed a method for the trajectory tracking of mobile robots based on adaptive control. Literature[5] designed a robust H-Infinite trajectory tracking controller for uncertain nonholonomic mobile robots,where the tracking error equation is transformed into linear variable parameter equation by using variable gain convex decomposition technique, and the state feedback controller is designed by using LMI theory to realize trajectory tracking. In reference [6], a asymptotic stability controller with diamond output constraints for the differential drive robot was designed under the condition of limited wheel torque.In reference [7], a feedback controller was designed to solve the trajectory tracking problem by adjusting the design parameters for meet the constraint conditions under the condition that the speed saturation limit of the robot was also affected by stall. Literature [8] designed a mobile robotic non-coupling linear PID trajectory tracking controller, and using the Lyapunov direct method to prove the system half global gradual stability.

However,all the above articles are assumed that the mass center and the geometrical center of robots is coincide, due to robot hardware structures, weight distribution, etc.,the mass center and the geometrical center are not coincide usually[9].This paper conducted research on trajectory tracking control issues in the case of the mass center and the geometrical center are not coincide， and designed a control protocol to make differential drive robots can track the reference trajectory.

2. Problem formulation

As illustrated in Figure 1-1, the two rear wheels are independent drive wheels, and the speed of the wheel is changed by adjusting the input voltage of each drive motor to form a variable change in the position of the vehicle body position.

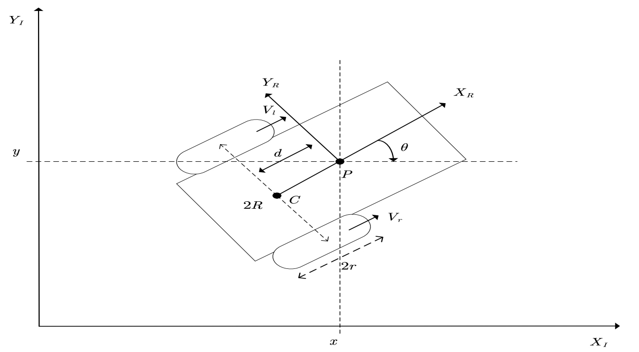

Fig. 1 Model of differential drive robot

As Illustrated in Fig.1, r denotes the radius of the wheel, d denotes the distance between the mass center and the geometrical center, θ denotes the angle of robot and the x-axis of the world coordinate system, point $P(x,y)$ denotes the coordinate of robot.

Suppose that the line velocity v of the robot is along the trajectory, the angular velocity is ω .The two drived wheels are purely rolled without sliding motion, and the robot is subject to the following nonholonomic constraints:

$$\dot{y}\cos\theta - \dot{x}\sin\theta - d\dot{\theta} = 0 \tag{2-1}$$

This can be obtained from a nonholonomic constrained robot's kinematics model:

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{2-2}$$

For the trajectory tracking problem of nonholonomic constrained wheeled robots, a virtual robot could be adopted as a reference, and then trajectory tracking problem could be transformed into a vritual leader tracking problem.When the robot's position, speed, direction are exactly same as the virtual robots, it could be deemed that the reference trajectory has been tracked.

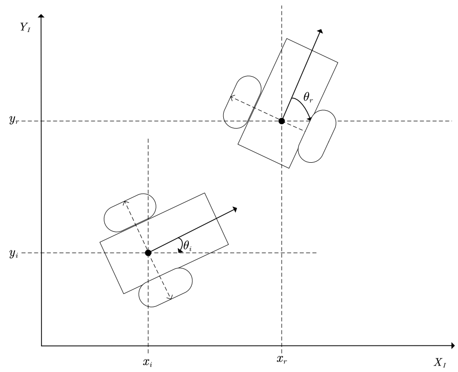

Fig .2 Illustration of error of robot tracking

As shown in Fig.2, (x_r, y_r, θ_r) is the position of a virtual robot, and (x_i, y_i, θ_i) is the position of the follower, and the reference trajectory is described by the virtual robot's motion trajectory,so the virtual leader's kinematics model could be described as:

$$\dot{\xi}_r = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -d\sin\theta_r \\ \sin\theta_r & d\cos\theta_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \tag{2-3}$$

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According to the location relationship between the virtual robot and the tracking robot, their tracking error could be described as the following form:

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix} \tag{2-4}$$

As long as there is a finite time in which the conditions between two robots satisfies the following relation, then the robot is said to have achieved the tracking of the target trajectory:

$$\begin{cases} \lim\limits_{t \to T} (x_r - x) = 0 \\ \lim\limits_{t \to T} (y_r - y) = 0 \\ \lim\limits_{t \to T} (\theta_r - \theta) = 0 \end{cases} \tag{2-5}$$

The derivative of formula(2-4) is :

$$\begin{cases} \dot{x}_e = \omega y_e - v + v_r \cos\theta_e - d\omega_r \sin\theta_e \\ \dot{y}_e = -\omega x_e + v_r \sin\theta_e - d\omega + d\omega_r \cos\theta_e \\ \dot{\theta}_e = \omega_r - \omega \end{cases} \tag{2-6}$$

Equation(2-6) is the closed-loop tracking error of the system , nonholonomic constraint trajectory tracking problem could be transformed into the stabilization problem of tracking error model as long as an appropriate control input $u = [v \ \omega]^T$ is designed. For any initial state error,the tracking error is tending to zero under the action of control input, which satisfy the following conditions:

$$\lim_{t \to T} \| (x_e \ y_e \ \theta_e) \| = 0 \tag{2-7}$$

3. Controller Design

Lemma 3-1 :(Barbalat lemma) if the differentiable function $f(t)$ is uniformly continuous, $\lim\limits_{t \to \infty} \int_0^t f(t)dt$ exists and is bounded, then $\dot{f}(t) \to 0$ when $t \to \infty$.

Theorem 3-2: For the control inputs v and ω of the system, if they are continuous and bounded in the interval $[0, +\infty)$, the design of the control protocol shown in (3-1) can make the closed-loop system (2-6) reach consensus and stability. In addition, if ω_r does not approach 0, the closed-loop system is said to be globally uniformly asymptotically stable.

Based on the above lemma, the following control inputs are selected for the tracking error system. Under the action of the control law, the system can be globally uniformly asymptotically stable:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_e + k_1 d\omega \sin\theta_e - k_2 \theta_e w - x_e + d\cos\theta_e - d \\ v_r \sin\theta_e + \omega_r \end{bmatrix} \tag{3-1}$$

Where k_1 and k_2 are normal numbers used to adjust the controller.

Proof: In order to prove the stability of tracking error closed-loop system, the following Lyapunov function is selected:

$$V(x_e, y_e, \theta_e) = \frac{1}{2}(x_e - d(\cos\theta_e - 1))^2 + \frac{1}{2}(y_e - d\sin\theta_e + \theta_e)^2 \tag{3-2}$$

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Obviously $V(0)=0$ and $V(\infty)=\infty$, so V is positive definite and radially unbounded. The derivative of (3-2) can be obtained:

$$\begin{aligned}\dot{V}=&(x_e+d(1-\cos\theta_e))\,(v_r\cos\theta_e-v+d\omega\sin\theta_e-\theta_e w)\\&+(y_e-d\sin\theta_e+\theta_e)\,(v_r\sin\theta_e+\omega_r-\omega)\end{aligned}\tag{3-3}$$

Substitute the control law (3-1) into the above equation:

$$\dot{V}(x_e,y_e,\theta_e)=-(x_e+d(1-\cos\theta_e))^2\tag{3-4}$$

Obviously $\dot{V}(x_e,y_e,\theta_e)\leqslant 0$, and the equal sign is valid when $x_e=0$, $\theta_e=0$. According to Lyapunov stability theorem, the system is globally uniformly stable at the equilibrium point, so it can be known that $x_e(t)$, $y_e(t)$, $\theta_e(t)$ is uniformly bounded on the interval $[0,+\infty)$. Furthermore, since $\dot{V}\leqslant 0$, V are positively definite, $V(t)$ is non-increasing under the control input and has a lower bound.Due to

$$\int_0^t\dot{V}(x_e,y_e,\theta_e)dt=V(x_e(t),y_e(t),\theta_e(t))-V(x_e(0),y_e(0),\theta_e(0))\tag{3-5}$$

so we can get

$$\lim_{t\to\infty}\int_0^t\dot{V}(x_e,y_e,\theta_e)\,dt\leqslant\delta\tag{3-6}$$

where δ is a bounded positive real number. And because of

$$\ddot{V}(x_e,y_e,\theta_e)=-2(x_e+d(1-\cos\theta_e))\left(\dot{x}_e+d\dot{\theta}_e\sin\theta_e\right)\tag{3-7}$$

We can get that $x_e(t)$, $y_e(t)$, $\theta_e(t)$ are bounded, and also the $v_r(t)$, $\omega_r(t)$ are bounded. Combined with the tracking error model (2-6), it shows that $\dot{x}_e(t)$, $\dot{y}_e(t)$, $\dot{\theta}_e(t)$ are also bounded, thus $\ddot{V}$ is also bounded. According to Lemma 3-1(Barbalat lemma), it can be obtained:

$$\lim_{t\to\infty}\dot{V}=\lim_{t\to\infty}\left[-(x_e+d(1-\cos\theta_e))^2\right]=0\tag{3-8}$$

We can get that $\lim\limits_{t\to\infty}x_e(t)=0$, $\lim\limits_{t\to\infty}\theta_e(t)=0$,it means within control input (3-1), the tracking error on x-anis and heading angle can converge to 0. In addition, it is necessary to prove that the tracking error on y-axis can converge to 0.

Substitute the control law into the first equation of the tracking error model:

$$\begin{aligned}\dot{x}_e=&(v_r\sin\theta_e+\omega_r)y_e-k_1d\omega\sin\theta_e+k_2\theta_e w\\&+x_e-d(\cos\theta_e-1)-d\omega_r\sin\theta_e\end{aligned}\tag{3-9}$$

due to $\lim\limits_{t\to\infty}\theta_e(t)=0$, $\lim\limits_{t\to\infty}x_e(t)=0$,so formula(3-8) could be simplified as

$$\omega_r y_e=0\tag{3-10}$$

then we can get that if $\omega_r\neq 0$,then $\lim\limits_{t\to\infty}y_e(t)=0$,so under the action of control law (3-1), the system is globally uniformly asymptotically stable, and the final tracking error will converge to 0.

4. Simulation result

In order to verify the effectiveness of the control law proposed in this article, the trajectory tracking results in the case of constant reference input and time-varying reference input are verified respectively.

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Track the constant reference trajectory

The initial pose of the reference robot is $(0, 0, \pi/4)$, reference velocity is $v_r = 2m/s$, angular velocity is $\omega_r = 1rad/s$, and the initial pose of the mobile robot is $(2, 3, \pi)$. The controller parameters are $k_1 = 2, k_2 = 0.5, d = 0.25$. The tracking trajectories are shown in figure 3.4 to 3.6:

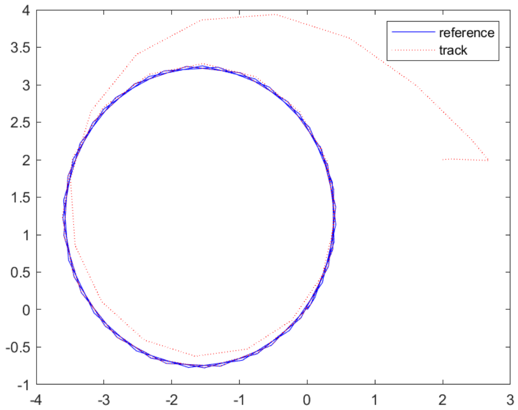

Fig3.4 Simulation results of trajectory tracking

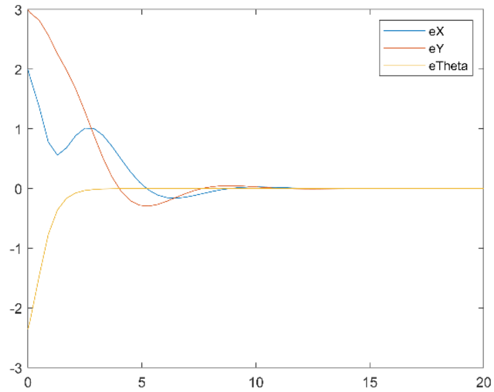

Fig3.5 Tracking error convergence

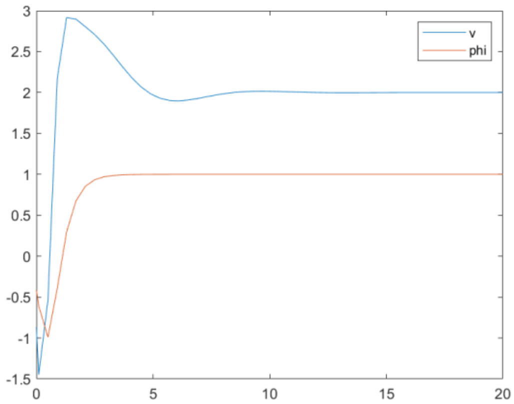

Fig3.6 The speed of the mobile robot

Track the time-varying reference trajectory

The initial pose of the reference robot is $(5, 5, \pi/2)$, reference velocity is $v_r = \sqrt{1 + \sin^2(t)}$, angular velocity is $\omega_r = -\cos(t)/(1 + \sin^2(t))$, and the initial pose of the mobile robot is

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$(0, 0, \pi)$. The controller parameters are $k_1 = 0.1$, $k_2 = 1$, $d = 0.25$. The tracking trajectories are shown in figure 3.7 and figure 3.8:

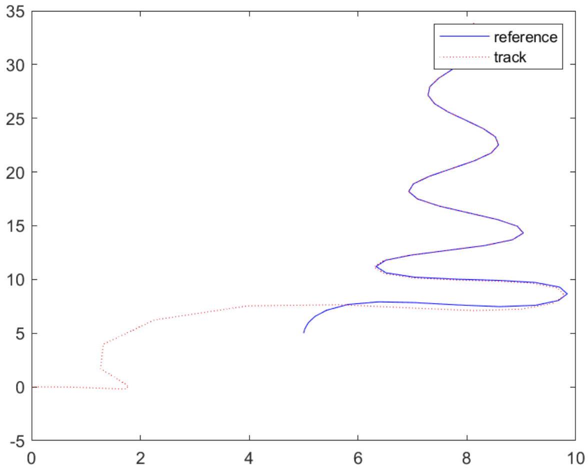

Fig3.7 Simulation results of trajectory tracking

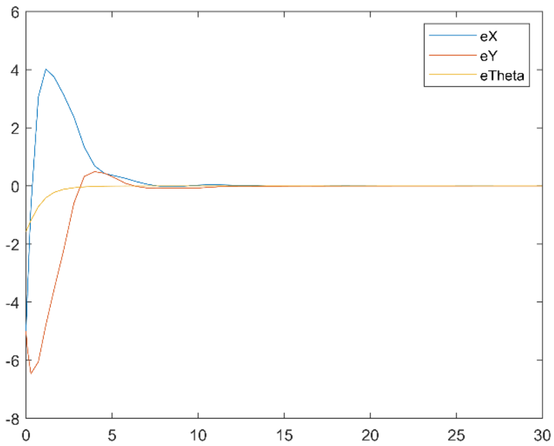

Fig3.8 Tracking error convergence

5. Conclusion

In this paper, a continuous time-varying trajectory tracking controller is designed for a nonholonomic constrained robot whose mass center and the geometrical center is not coincide. Lyapunov stability theorem and Barbalat theorem are used to prove that the control algorithm is globally uniformly asymptotically stable. Finally, the effectiveness of the control algorithm is verified by simulation.

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