ISSN: 2664-9640

Solution of Vehicle Routing Problem Model with Shortest Production Cycle in Multi-depot

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Abstract

Aiming at the vehicle routing problem with the shortest production cycle in multi-depot, a multi-constrained vehicle routing model including product output, multi-depot and customer point capacity is established, and an improved ant colony algorithm is designed to solve the model. In the process of pheromone global update and probability transfer rule, the algorithm uses a specific heuristic function and changes the evaluation method of the optimal solution in the iterative process. The simulation results show that the algorithm is superior to the traditional ant colony algorithm and the comparative genetic algorithm in terms of convergence speed, convergence accuracy and optimal solution quality in the process of optimizing the objective function, which verifies the effectiveness of the model and algorithm. Considering the shortest production cycle scheduling scheme, the average total time is reduced by 6%, which can provide a more reasonable vehicle distribution scheme for enterprises.

Keywords

Vehicle Routing Problem; Optimization scheduling; Optimal path; Ant colony algorithm.

1. Introduction

Vehicle Routing Problem, The term VRP was first put forward by Dantzig and Ramse in the late 1950s. After continuous development and evolution, it has become the research focus of operational research, graph theory, Internet of Things and other disciplines. The basic idea of vehicle optimal scheduling is: dispatch the motorcade to deliver services to customers with different demand for goods, and plan appropriate routes for the motorcade to complete the whole delivery task while observing certain constraints. It can be simply described as that the distribution vehicle starts from one (or more) distribution centers, passes through several distribution points (randomly distributed) in a certain order under various constraints, and completes all distribution tasks.

Vehicle routing problem can be divided into many types according to its different problems. Generally, the vehicle routing problem aims at minimizing the total journey (total running time) or total cost of all vehicles, while in some cases, it is often necessary to consider various production objectives such as the minimum cost and the shortest production cycle. Due to the different research angles of practical problems, different scholars have added various specific constraints to traditional VRP problems, resulting in many new problems. Aiming at the multi-depot heterogeneous vehicle routing problem[1], Salhi et al. The multi-vehicle routing problem with picking operation is studied, and a heuristic algorithm based on set partition is proposed to solve it [2]. Sundar et al. Aiming at the problem of multi-depot and multi-vehicle with fuel restriction, a branch cutting algorithm is designed, but this algorithm is only suitable for small-

scale situations[3]. Luo et al. The extremum optimization algorithm is introduced into the hybrid Shuffled Frog Leading Algorithm to study the vehicle routing problem with time windows in multiple parking lots[4]. Dayarian et al. Taking the milk collection system as an example, a branch pricing method is proposed to study the multi-attribute vehicle routing problem[5]. Calvet et al. Considering the potential influence of different warehouse distribution on demand change, a hybrid method combining statistical learning technology with meta-heuristic framework is designed[6]. De Oliveira et al. The MD-VRP problem is decomposed into several classical VRP subproblems, and a parallel evolutionary algorithm with variable length genes and search operators is proposed. The effectiveness of the proposed algorithm is verified by an example[7]. Calvet et al. This paper studies the multi-depot problem with limited fleet, that is, limited warehouse capacity, and proposes a simulation heuristic framework combining Monte Carlo simulation and meta-heuristic algorithm[8]. Yang Ye et al. The multi-vehicle full-load vehicle scheduling problem with time windows is studied, and a heuristic algorithm of "scan-insert-genetic" is designed. However, the obtained solution is very dependent on the scanning situation, and it is often impossible to get the global optimal solution[9]. Yong Ye et al. Considering the dynamic activation of distribution center, an improved wolf pack algorithm is designed[10]. Hongyu Ma et al. The driver's salary, including basic salary and overtime pay, is introduced to study the multi-depot and multi-model vehicle scheduling problem with the minimum delivery fee as the goal. However, in the process of algorithm verification, the case scale is small, and the multi-model is simplified into a single model, which reduces the difficulty of solving, and the result is not convincing enough[11]. Lijiao Liu et al. Carbon emissions are introduced into the multi-depot and multi-vehicle routing problem, and a single-cycle structure comprehensive learning bacterial foraging optimization algorithm is proposed. Its effectiveness is verified by single-peak function and multi-peak function, and the advantages of multi-depot and multi-vehicle over single-depot and single-vehicle are proved[12]. Dongdong He et al. A multi-vehicle green vehicle path model with time window is established, and an improved tabu search algorithm is designed[13]. Baradaran et al. Based on the actual vehicle distribution system, the multi-vehicle routing problem with multiple hard time windows with priority is studied, and three multi-objective models are proposed, which are verified by binary artificial bee colony algorithm[14]. Xiang Yang et al. Aiming at the open multi-depot vehicle routing problem under fuzzy demand, a two-stage tabu search algorithm is adopted to study it[15]. For multi-center distributed enterprises, when scheduling vehicles, we should not only consider the needs of the customers we serve, but also comprehensively consider the locations of multiple parking lots, product costs and the attributes of different types of vehicles. Compared with ordinary VRP problems, the difficulty of solving these problems is greatly increased. At present, there is still no mature and effective solution algorithm.

Therefore, on the basis of it, this study further increases the complexity of the model, aiming at minimizing the production cycle, and puts forward a vehicle routing problem model with the shortest production cycle in multi-depot, considering the capacity of demand points and different locations of depot. The improved ant colony algorithm is designed to solve the problem, and the effectiveness of the model and algorithm is verified by an example.

2. Description and Modeling of Vehicle Routing Problem with Shortest Production Cycle in Multi-depot

2.1. Problem Description

The vehicle routing problem with the shortest production cycle in multi-depot is described as: there are d depots, each depot has its own vehicle v_l with capacity of w, l= 1,2, .. d vehicles, a total of m vehicles, m=$\sum_{l=1}^{d} v_l$. Responsible for the distribution of goods to n customer points.

The demand for goods at customer point i is $q_{in}(i = 1.., n)$, and $q_i < w$. The distribution process diagram is shown in Figure 2.

The distance between cars and customers and between customers is different, so the driving time is different. Regardless of the influence of road conditions and human factors, the average time of normal driving represents the driving time of two places, and the driving time from each yard to each customer is recorded as $td_{ki}, k = 1,2,..d, i = 1,2,..n$, The driving time from the i customer to the j customer is recorded as t_{ij}, $i = 1,2,..n, j = 1,2,..n$, All times satisfy the triangular inequality relation. Meet the requirements by reasonably arranging the service customer group and service order of each car. Only when every customer gets the service can the whole task be completed, so the completion time is the time required by the last customer who gets the service. Obviously, the last customer who gets service must be the last customer of a certain vehicle, and the arrival time of the last customer of each vehicle is the completion time of the vehicle, regardless of the time taken to return to the parking place from the last customer, let the completion time of each vehicle be $T_k, k = 1,2,..m$, Assume that the parking lot of the k car serves l_0, and the customer order is $i_1, i_2,..i_j$. The completion time of the whole production cycle is the completion time of the last vehicle, and the task completion time is r.The more vehicles, the fewer customers each car serves, and the shorter the completion time. However, the number of vehicles is limited. The fastest vehicle routing problem in multi-yard is to give a reasonable vehicle routing arrangement to make the completion time the shortest under the condition that the number of vehicles used in each yard does not exceed the number of vehicles it owns.

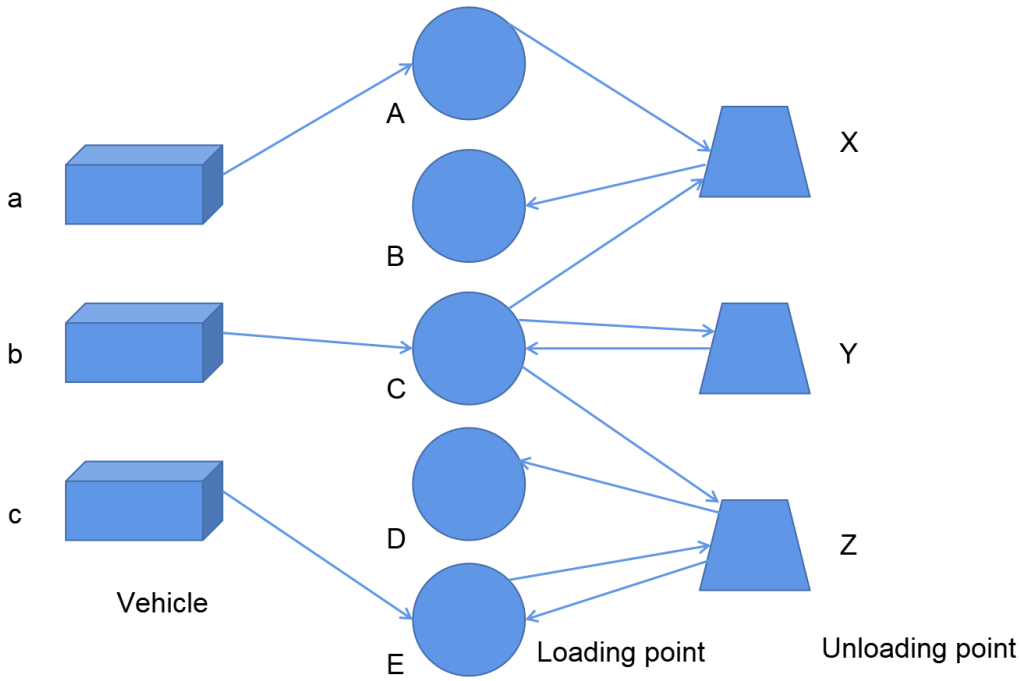

Fig. 1 Schematic diagram of vehicle routing problem

2.2. Model Building

2.2.1. Assumed Condition

(1) All vehicles must start from the distribution center and return to the distribution center after the service is completed.

(2) Every car is fully loaded.

(3) Vehicles in the distribution center have the same maximum load and drive at the same speed.

(4) Each customer is served by only one vehicle.
(5) There are capacity restrictions in the central yard.
(6) There is a capacity limit for customers.
(7) One car can visit multiple customers.
(8) The shortest time to complete the task.
(9) The vehicle can't be maintained during work, and it always runs normally.

2.2.2. Symbol Description

The basic symbol description is shown in Table 1.

Table 1 Parameter definition of model

Name	definition
d	Total number of car parks
w	Vehicle capacity
m	Total number of vehicles
n	Total customers
k	Park number
v_l	Number of vehicles in each Park
i	Customer point number
q_i	Customer demand
y_{ki}	Service from vehicle k to customer i
x_{kij}	k vehicles from i to j
td_{ki}	Travel time from each park to customers
t_{ij}	Travel time from customer i to customer j
T_k	Completion time of each vehicle
T	Task completion time
f_i	Customer point capacity
g_k	Park freight demand

2.2.3. Mathematical Model

According to the above problem description research, the objective optimization problem model will be established. The model takes the shortest time for the vehicle to complete the task as the objective function, and the influencing factors related to the model are reflected in the constraint conditions. The model is constructed as follows:

$$\mathrm{T} = \min \sum_{k=1}^{m} T_k \tag{1}$$

$$\text{s.t. } \sum_{k=1}^{n} T_k q_i \leq f_i \tag{2}$$

$$\sum_{i=1}^{n} y_{ki} \leq g_k \tag{3}$$

$$\sum_{k=1}^{m} y_{k0} = m. \tag{4}$$

$$\sum_{k=1}^{m} x_{kij} = y_{ki}. \tag{5}$$

$$y_{ki} = 1,0. \tag{6}$$

$$x_{kij} = 1,0. \tag{7}$$

The objective function (1) requires the shortest transportation time. (2) The transportation volume of customer point I cannot exceed the capacity of that point; Formula (3) indicates that the single delivery amount of each vehicle cannot exceed its maximum carrying capacity; Formula (4) indicates that the number of vehicles used by the dispatching center does not exceed the total number of available vehicles in each distribution center; Type (5) The task can

only be served once by one car; Formula (6) and (7) indicate that vehicles cannot travel from one distribution center to another.

3. Literature References

3.1. Fundamental Principle

Ant Colony Optimization was put forward by Professor Dorigo in 1992. It is a self-organizing, systematic and distributed positive feedback algorithm, which chooses the path according to the concentration of pheromone. The probability $p_{ij}^{k}(t)$ of choosing a city is as follows:

$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum_{j\in N_i^k}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}\\ 0, j\in N_i^k\end{cases} \tag{8}$$

In which: τ_{ij} represents the pheromone concentration between customers i and j ; η_{ij} represents the heuristic information between cities i and j; α indicates the relative importance of pheromone in the movement of ants. When α= 0, the movement mode of ants will follow the heuristic information, that is, the cities that ants will choose only consider the distance between two cities. β indicates the relative importance of heuristic information in the movement of ants. When β =0, the movement of ants will be carried out according to pheromone, which will cause the path of ant colony to be fixed and it is difficult to find the optimal solution. From the above, it can be seen that the main influencing factor of ant colony's selection of cities is the pheromone concentration between cities. If the pheromone concentration is too low, it will lead to a longer operation time. If the pheromone concentration is too high, it will accelerate the convergence and the global optimal solution will not be obtained. Therefore, the performance of the algorithm is improved by optimizing the algorithm.

3.2. Improvement Project

(1) Parameter initialization: an ant is randomly placed in each city, that is, the size of the city is the same as the number of ants. In the iterative process of the algorithm, a node is randomly selected as the starting point each time, which can effectively shorten the search time.

(2) State transition rule: the transition probability of ants may suddenly change before selecting the next node. If there is a sudden change, the transition probability to the line with small pheromones will increase from small to large, and the transition probability to the line with large pheromones will decrease from large to small; If there is no mutation, the probability of choosing a path with low pheromone concentration is small, and the probability of choosing a path with high pheromone concentration is high. In this way, ant colony algorithm is more likely to find the global optimal solution.

(3) Self-adaptive adjustment of pheromone volatilization coefficient: set the initial value of volatilization coefficient ρ as $\rho(t_0)$. When the number of iterations reaches N times, calculate the sum of absolute values of the difference between the optimal solution of the last five times and the optimal solution of the previous generation and compare it with the set threshold. If it is greater than the set threshold, the volatilization coefficient ρ will remain unchanged. If it is less than the set threshold, adjust the volatilization coefficient according to the following formula (9):

$$\rho(i+1)=\begin{cases}\mu\rho(i), & \rho(i+1)\geq\rho_{min}\\ \rho_{min}, & \rho(i+1)\leq\rho_{min}\end{cases} \tag{9}$$

Based on the above definition of improved ant colony algorithm, its mathematical model and improvement ideas, the steps of improved ant colony algorithm based on traveling salesman problem (TSP) are as follows:

STEP1: Set NC=1(NC is the number of searches or iterations), and initialize the variables $\tau_{ij}(0)$ 、Q、α、β 、ρ and $\Delta\tau_{ij}^{k}(t)$, m , n, etc., where m= n;

STEP2: Transforming the urban distribution map into an adjacency matrix map, placing m ants on n adjacency matrix coordinate points, storing the initial starting point of each ant in the defined solution set, and establishing $tabu_k$ set; For each ant $k(k = 1,2, \dots m)$, the probability $p_{ij}^{k}(t)$ is calculated according to the heuristic function and pheromone, and the next coordinate point j of the ant is selected according to the $p_{ij}^{k}(t)$ (where the probability of $p_{ij}^{k}(t)$ is abrupt, and the ant may choose the path with lower probability), so as to reach the coordinate point.

STEP3: If all cities have been put into the $tabu_k$ set, calculate and record the current best solution and the best path;

STEP4: Global update of pheromone of each path (i, j) according to formula (10)$\tau_{ij}(t+1) = (1-\rho)\cdot\tau_{ij}(t) + \Delta\tau_{ij}(t)$ or formula (11)$\tau_{ij}(t+1) = (1-\rho)\cdot\tau_{ij}(t) + \rho\Delta\tau_{ij}(t)$;

STEP5: For each path (i, j), let $\Delta\tau_{ij}^{k} = 0$ and NC = NC+1;

STEP6: When NC≥50, calculate the sum of the absolute values of the difference between the optimal solution of the last five times and the optimal solution of the previous generation. If it is greater than the set threshold, the pheromone volatilization coefficient will remain unchanged; if it is less than the set threshold, execute formula (9) and adaptively change the pheromone volatilization coefficient.

STEP7: If NC is less than the predetermined number of iterations, set the tabu table to idle STEP2 to continue traversing, otherwise, move to the next step;

STEP8: Stop traversing, output the current optimal solution, and stop the calculation.

3.3. Algorithm Parameter Definition

The parameter definitions of the improved ant colony algorithm designed in this paper are shown in Table 2.

Table 2 Algorithm parameter definition

Name	definition
τ_{ij}	Pheromone concentration between customers i and j
Q	Pheromone concentration constant
ρ	Volatilization coefficient
NC	Iterations
α	Pheromone important factor
β	Heuristic function important factor
td_{ki}	Volatilization degree of pheromone
t_{ij}	Path update pheromone
$tabu_k$	Taboo list
$\Delta\tau_{ij}^{k}$	Pheromone increment
m	Ant number
n	Number of tasks

The algorithm has the following advantages:

(1) The whole process of the algorithm is a positive feedback mechanism, which makes the search process continuously converge and approach the optimal solution;

(2) Adding heuristic rules effectively overcomes the problem that traditional ant colony algorithm is easy to fall into local optimum;

(3) Adding the rule of weighted proportion of accessories to the probability transfer rule can make the first optimization objective converge quickly.

4. Simulation Experiment And Results

4.1. Simulation Experiment

Taking the transportation of a production enterprise as an example. The goods produced by the production workshop are piled up at the loading point and transported to different warehouses respectively, and the motorcade is sent between the loading point and the warehouse for circular transportation. Among them, there are 5 loading points (denoted by letters A-E), 3 warehouses (denoted by letters X-Z) and 3 vehicles (denoted by numbers a-c), and their detailed information is shown in Table 3.

Table 3 Location and demand information table

Number	Coordinate/km	Weight/ton
A	[2,2]	20
B	[1,4]	30
C	[3,5]	40
D	[5,3]	50
E	[7,5]	60
X	[4,1]	60
Y	[6,1]	70
Z	[8,1]	80
a	[1,1]	10
b	[3,3]	10
c	[8,5]	10

Assuming that each train is fully loaded, according to the capacity of the vehicle, the loading point to the unloading point is regarded as a task. There are several such tasks, and the numbers are 1,2,3 …. The distance between each task is shown in Table 4. As the vehicle runs at a constant speed, the driving time can be obtained according to the distance, and then the target optimized route can be obtained according to the improved ant colony algorithm.

Table 4 Distance information table between tasks

Number	1	2	3	4	5	6	7	8	9	10	…
1	6	6	5	5	5	5	3	6	3	3	…
2	6	6	5	5	5	5	3	6	3	3	…
3	6	6	5	5	5	5	3	6	3	3	…
4	6	6	5	5	5	5	3	6	3	3	…
5	6	6	5	5	5	5	3	6	3	3	…
6	6	6	5	5	5	5	3	6	3	3	…
7	8	8	7	7	7	7	5	8	3	3	…
8	8	8	7	7	7	7	5	8	3	3	…
9	8	8	7	7	7	7	5	8	3	3	…
10	8	8	7	7	7	7	5	8	3	3	…
…	…	…	…	…	…	…	…	…	…	…	…

ISSN: 2664-9640

4.2. Experimental Result

According to Table 5, the probability of the improved ant colony algorithm to get the target value around 500km is 70%, and the worst solution of 10 experiments is 340 km, which is still better than the optimal solution of the basic ACO algorithm. It shows that the improved ant colony algorithm has greatly improved the performance, and the overall quality of the obtained solution completely exceeds that of the basic ant colony algorithm, which can well handle the problem of the shortest production cycle of multi-depot. The optimal solution of the 210km vehicle route is as follows:

Routing1: 3—9—12—11—8—6—2

Routing1: 7—16—19—20—10—5—4

Routing1: 14—18—17—15—13—1

The minimum number of iterations is 51. Compared with the results in reference, the optimal solutions are all 423, and the minimum number of iterations of the improved algorithm in this paper is 51, which is better than 67 in reference0. The comparison chart of experimental results is shown in Table 5:

Table 5 Comparison of computational experiment results

	Improved ACO	Traditional ACO	Comparative GA
average value	230	300	500
optimal value	210	290	470

In this paper, the improved ant colony algorithm will be adopted, and the change of the average value in the iterative process is shown by the red dotted line in Figure 2, and the value of the objective function corresponding to the optimal solution is shown by the blue solid line in Figure 2.

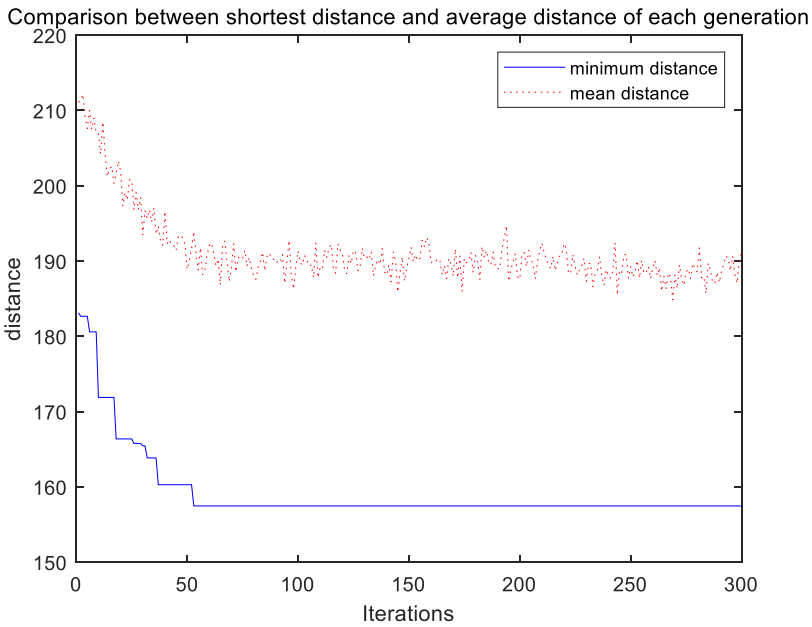

Fig. 2 Mean value of convergence curve of objective function one

From Figure 2 and Table 5, it can be seen that in the process of optimizing the first objective function, that is, the shortest production cycle vehicle routing problem, compared with the traditional ant colony algorithm, the improved ant colony algorithm converges faster.

5. Summary

In order to solve the vehicle routing problem in the transportation process of today's production enterprises, an improved ant colony algorithm combining roulette operation and mutation optimization algorithm is proposed. Compared with the traditional ant colony algorithm, it is found that the improved ant colony algorithm improves the performance by about 11% and has stronger stability. Compared with ant colony algorithm and improved ant colony algorithm, the accuracy of solution is also greatly improved. Therefore, the proposed algorithm can better solve the vehicle routing optimization problem with minimum production cycle in multi-depot, and better save the logistics cost for the corresponding enterprises.

Acknowledgements

The authors acknowledge the support from China Postdoctoral Science Foundation (2017M623061), Natural Science Foundation of Hunan Province (2020JJ4724) and National Engineering Research Center for Oil & Gas Drilling Equipment (2021-2.3)

References:

[1]. Salhi, SM Sari. A multi-level composite heuristic for the multi-depot vehicle fleet mix problem. European Journal of Operational Research. Vol. 103 (1997) No. 1, p. 95-112.
[2]. Viera., Idgiosaand. New Assignment Algorithms for the Multi-Depot Vehicle Routing Problem. The Journal of the Operational Research Society. Vol. 53 (2002) No. 9, p. 977-984.
[3]. Eusuff, MuzaffarmKevine Lansey. Optimization of Water Distribution Network Design Using the Shuffled Frog Leaping Algorithm. Journal of Water Resources Planning and Management. Vol. 129 (2003) No. 3, p. 210-225.
[4]. Salhi, Said, Niaz WassanMutaz Hajarat. The Fleet Size and Mix Vehicle Routing Problem with Backhauls: Formulation and Set Partitioning-based Heuristics. Transportation Research Part E: Logistics and Transportation Review. Vol. 56 (2013) 22-35.
[5]. Luo, JianpingMin-rong Chen. Multi-phase modified shuffled frog leaping algorithm with extremal optimization for the MDVRP and the MDVRPTW. Computers & Industrial Engineering. Vol. 72 (2014) 84-97.
[6]. Luo, Jianping, Xia LiMin-rong Chen, et al. A novel hybrid shuffled frog leaping algorithm for vehicle routing problem with time windows. Information Sciences. Vol. 316 (2015) 266-292.
[7]. Dayarian, Iman, Teodorgabriel CrainicMichel Gendreau, et al. A column generation approach for a multi-attribute vehicle routing problem. European Journal of Operational Research. Vol. 241 (2015) No. 3, p. 888-906.
[8]. Bernardes De Oliveira, Fernando, Rasul EnayatifarHosseinjavedani Sadaei, et al. Erratum to "A cooperative coevolutionary algorithm for the Multi-Depot Vehicle Routing Problem [Expert Systems with Applications 43 (2015) 117–130]". Expert Systems with Applications. Vol. 54 (2016) 398-402.
[9]. Calvet, Laura, Albert FerrerMisabel Gomes, et al. Combining statistical learning with metaheuristics for the Multi-Depot Vehicle Routing Problem with market segmentation. Computers & Industrial Engineering. Vol. 94 (2016) 93-104.
[10]. Calvet, Laura, Dandan WangAngel Juan, et al. Solving the multidepot vehicle routing problem with limited depot capacity and stochastic demands. International Transactions in Operational Research. Vol. 26 (2019) No. 2, p. 458-484.
[11]. Arnold, FlorianKenneth Sörensen. Knowledge-guided local search for the vehicle routing problem. Computers & Operations Research. Vol. 105 (2019) 32-46.
[12]. Baradaran, Vahid, Amir ShafaeiAmirhossein Hosseinian. Stochastic vehicle routing problem with heterogeneous vehicles and multiple prioritized time windows: Mathematical modeling and solution approach. Computers & Industrial Engineering. Vol. 131 (2019) 187-199.
[13]. Arnold, FlorianKenneth Sörensen. What makes a VRP solution good? The generation of problem-specific knowledge for heuristics. Computers & Operations Research. Vol. 106 (2019) 280-288.

[14]. Bac, UgurMehmet Erdem. Optimization of electric vehicle recharge schedule and routing problem with time windows and partial recharge: A comparative study for an urban logistics fleet. Sustainable Cities and Society. Vol. 70 (2021) 102883.
[15]. Konstantakopoulos, Grigoriosd, Sotirisp GayialisEvripidisp Kechagias, et al. A Multiobjective Large Neighborhood Search Metaheuristic for the Vehicle Routing Problem with Time Windows. Algorithms. Vol. 13 (2020) No. 10, p. 243.
[16] Yu, Lei, Shi, Feng, Wang, Hui, et al. Analysis of 30 Cases of MATLAB Intelligent Algorithm. Beijing University of Aeronautics Astronautics Press, 2015, p. 229-236.