ISSN: 2664-9640

The Longest Lasting Sandcastle and Its Sand-to-water Mixture Proportion

Lingnan Xu *

Department of Computer Science and Technology, North China Electric Power University, Baoding 071000, China

337439072@qq.com

Abstract

The paper aims to construct a model to find a better design of a sandcastle foundation that can resist the erosion of waves and tides. Initially, by calculating anti-erosion coefficients and comparing the drag characteristic of different streamlined bodies, we find that the combination of the semi-ellipse surface with arc streamlined surface is the best 3D shape of a sandcastle foundation and figure the reference parameters of it, which are h_{max}=20 cm, L_E=4 cm, L_R=12 cm, and D_0=16 cm. Meanwhile, based on the model, we analyze the adhesion and strength of the sand and find the relationship between the elastic shear modulus and the volume fraction of water, to determine the optimal sand-to-water mixture proportion as 49.68. Moreover, to improve the model, we comprehensively considered the erosion and infiltration of the top surface by rainfall, and calculated a reference value of θ = 68 ° as the best inclined angle. Furthermore, we analyze other factors, including the distance between the sandcastle and the edge of the beach, to improve the stability and erosion resistance of the foundation model by adding a threaded drain, building a moat, or changing the type of sand to river sand and finally construct a feasible and reasonable model which can accommodate to various situations.

Keywords

Sandcastle, Mathematical Modeling.

1. Introduction

1.1. Background

There are more and more sand beaches for people's entertainment and leisure in the world. Many children and adults will choose to make interesting sandcastles on the seashore. They will use a variety of tools and toys to give full play to their imagination and creativity, imitating the characteristics of real castles, such as walls, towers, moats, etc., with different degrees of complexity. However, casually built sandcastles are often unstable, so before creating a truly strong and lasting sandcastle, the foundation of a sandcastle is usually built to ensure its stability.

Figure 1: Sandcastle

Figure 2: An eroded sandcastle

However, many natural factors, such as the impact of waves, the rise of tides, the impact of rain, the influence of wind and the inner factors of sandcastles, will cause various degrees of erosion and damage to sandcastles. Different sand fortresses are affected by the external environment to different degrees. One quick example: any child playing on the beach knows that the physical properties of wet and dry sand are very different. Wet sand can be used to build sharp-featured sandcastles that would be unstable in dry sand. [1] Therefore, under the consideration of limiting factors, people hope to find an optimal three-dimensional geometry for the construction of sand fortress foundation, so that the duration of sand fortresses is longer and more substantial while meeting people's requirements to the greatest extent.

1.2. Restatement of the Problem

To identify the best 3-dimensional geometric shape to use as a sandcastle foundation, we are required to build a mathematical model. The problem is analyzed into four parts:

Build a model to simulate the inflow of ocean waves coupled with rising tides and predict the optimal geometry of foundation.

Use the model to determine the best sand water mixture ratio of the castle foundation, without using other materials.

Analysis model affected by rainwater.

Provide innovative schemes to make the sandcastle last longer.

1.3. Our work

We are asked to find out the best 3-dimensional geometric shape of sandcastle foundation, develop mathematical models to determine an optimal sand-to-water mixture proportion, furtherly analysis the model affected by rainwater, and put forward other strategies to make the sandcastle last longer.

Due to the restrictions in the requirements, the threat to the stability of sandcastle can be mainly attributed to the water erosion caused by tides and waves. Firstly, we divide the water erosion into three stages according to the stress condition and erosion effect. By analyzing the conservation of energy and capillary bridges between sand grains, we find the quantitative relationship between critical height and base radius. Based on a series of indexes, we conclude that the semi-ellipse with arc streamlined body has the best drag characteristic. Thus, we build curvilinear equations to work out the best sandcastle foundation model.

Secondly, through theoretical analysis about the adhesion and strength of the sand, we analyze the relationship between the elastic shear modulus and volume fraction of water. To further improve the accuracy of the model, we conduct the simulation experiment of wave impact and find that the experimental results about an optimal sand-to-water mixture proportion are in good agreement with the predicted values of the theory.

Thirdly, we considered the impact of rain erosion as one of the determinants. To improve our 3-dimensional model, we comprehensively considered the erosion and infiltration of the top surface by rainfall, basing on our former research. We find the relationship between superficial area and the radius of the top circle. Through analyzing the relationship between anti-erosion ability and slope inclination, we calculated the best inclined angle.

Fourthly, we considered other factors that can increase the stability and erosion resistance of the foundation, including the distance between the sandcastle and seawater, constructing the helix on the outer surface of the foundation to drain water, building a ‘moat’, and changing the type of the sand.

ISSN: 2664-9640

2. General Assumptions

Sandcastle foundation is built on the same sand.
The type of sand is the same, so is the seawater.
The acceleration of gravity is about 9.8 m/s^2.

3. Symbol Description

The following table provides meanings of important notations used in our models

Symbol	Meaning
g	The acceleration of gravity.
t	Any time.
H	Tide high at any time.
H_0	The correction of tide datum.
H_{max}	The highest water level.
m	The mass of a section of water column.
E_g	The gravitational potential energy.
E_k	The kinetic energy.
v_0	The velocity of tide when rushing to the beach.
v_1	The velocity of tide when hitting the front of the sandcastle.
D	The furthest distance of waves scouring the beach.
d	The distance between sandcastle and the edge of the sea.
Δv	The velocity loss of tide when hitting the front of the sandcastle
S_A	The surface area facing the tide.
L_0	The length of the transverse section of the base.
Δv	The velocity loss of tide.
h_{max}	The critical height of a sandcastle.
R	The circumradius of the bottom profile figure.
G	The elastic modulus.
ρ	Density.
S	The anti-erosion coefficient.

D_0	The maximum cross-sectional diameter of the sandcastle foundation.
G'	The shear modulus.
L_E	The length of the inflow section
L_R	The length of the outflow section
r_{rain}	The raindrop radius.
v_{ter}	The terminal velocity.

4. Model One – the Best 3D Geometric Shape Model

To identify the best geometric shape of a sandcastle foundation that will last the longest period time on a seashore experiencing waves and tides, a comprehensive model combining the knowledge of geography and physics is essential and inevitable. Thus, a model based on the conservation of energy, the elastic modulus of the wet sand, the semi-ellipse with arc streamlined body and curvilinear equations is established.

4.1. Assumptions

The sandcastle's infrastructure is at the same distance as the water on the same beach.

The type, quantity, and proportion of sand are the same.

The threat to the stability of sandcastle can be mainly attributed to the water erosion caused by tides and waves. Since wind erosion is a relatively long and insignificant process, we can ignore its role.

4.2. Establishing the Model

4.2.1 The Height of a Sandcastle Foundation

From the perspective of geography, the water in the middle of the flowing water has the fastest flow velocity, and the strongest hydrodynamic force will impact the river bank directly in front. This water flow will increase the local water pressure of the concave bank and form a spiral circulation (Figure 3). This will further erode the concave bank and pile up the eroded sand and gravel to the convex bank. Therefore, the erosion resistance of the convex edge is much stronger than that of the concave bank.

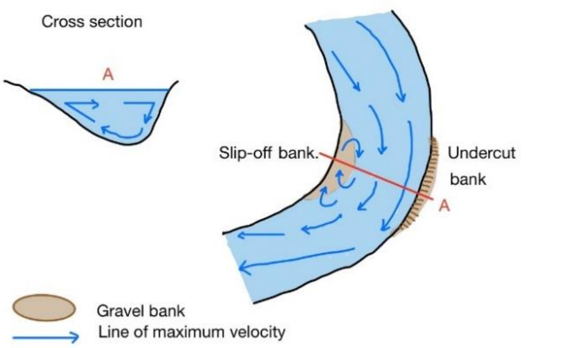


Figure 3: Slip-off bank and Undercut bank

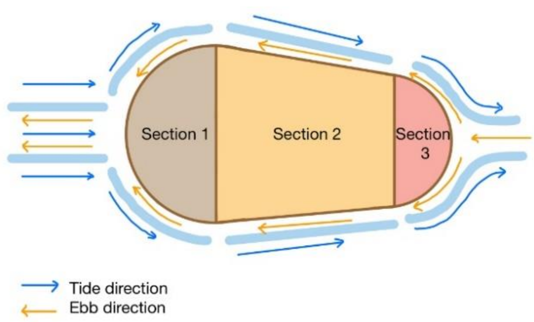


Figure 4: Partition (top view)

By analyzing the forces acting on the sandcastle, the wave erosion can be divided into three stages: erosion at high tide, side erosion, and erosion at ebb tide.

In the first stage, the front of the sandcastle will be first beaten by the waves and tides; in the second stage, the side of the sandcastle will be eroded by the water in the process of moving forward; in the third stage, the back of the sandcastle will be eroded by the tide returning to the farthest place. Therefore, because of these three stages, we simplify the model of sandcastle's foundation, divide it into three parts (Figure 4), and analyze the stress conditions respectively to get the optimal model.

To get the current velocity of the waves rushing to the beach, we sampled and analyzed the tide table of Sanya, a popular beach tourism city in China, on a certain day in different months of the year (Figure 5). We found that although there was a deviation in the time of tide rise and fall in different periods, for most months of the year, the time interval between the highest water level H_{max} and the time interval T of water level higher than sea level was approximately constant and satisfied the sinusoidal relationship:

$$H = H_{max} \sin \frac{2\pi}{T} t - H_0$$

where t denotes any moment, H denotes tide high at any time, H_0 denotes the correction of tide datum.

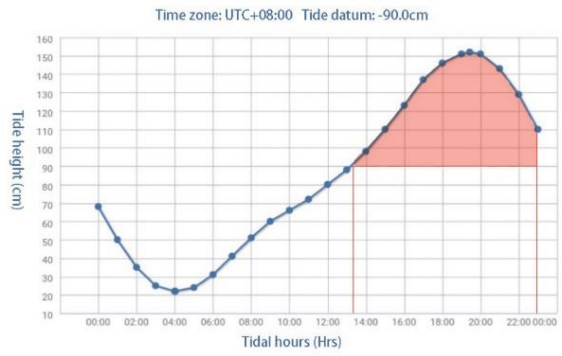


Figure 5: Tidal table (2020/3/6 Sanya China)

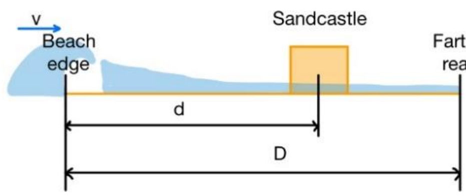


Figure 6: Distance ratio

We choose tide datum $H_0 = -90\ cm$, the average value $H_{max} = 60\ cm$, $T = 24\ h$ for Sanya. Assuming that the seawater is evenly distributed in the vertical plane, we take a section of the water column with a mass of $m\ kg$, and obtain the gravitational potential energy E_g at the highest water level:

$$E_g = \int_0^{H_{max}} \frac{mg}{H_{max}} h\, dh = \frac{mgH_{max}}{2}$$

Neglecting the energy loss, the tidal kinetic energy E_k when rushing to the beach:

$$E_k = \frac{1}{2} m v_0^2$$

Considering the conservation of energy, the velocity of the tide when rushing to the beach is:

$$v_0 = \sqrt{gH_{max}} = 2.42\ m/s$$

Let the furthest distance of waves scouring the beach be $D\ m$, and the sandcastle is built at $d\ m$ away from the edge of the sea (figure 6), and this can be roughly regarded as a uniform deceleration process. So, we have:

$$a = \frac{-v_0^2}{2D}$$

where a denotes the acceleration of tide.

The velocity of the tide when hitting the front of the sandcastle is v_1:

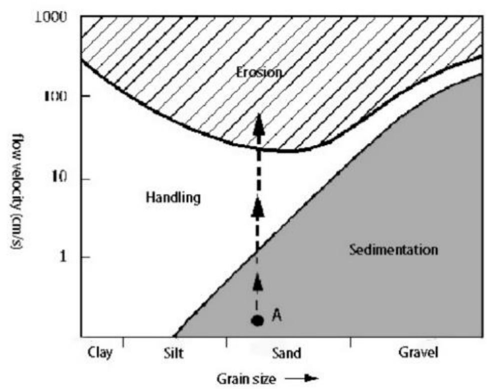
$$v_1 = \sqrt{v_0^2 + 2ad} = \sqrt{1 - \frac{d}{D}} v_0$$

As for the sandcastle foundation which is at the same distance as the water on the same beach and the type, quantity, and proportion of sand are the same. The velocity loss of tide Δv when hitting the front of the sandcastle has the following relationship with the surface area S_A facing the tide:

$$\Delta v = \frac{S_A}{2\pi R^2} v_1$$

where R denotes the circumradius of the bottom profile figure. The greater the positive velocity loss, on the one hand, it has a deceleration effect, on the other hand, it also means that the more energy the sandcastle bears, the more serious the erosion on the front of the sandcastle is.

Figure 7(The Hjulström curve) depicts the critical velocity of erosion, transport, and deposition shown by the change of sediment grain size and flow velocity. We can see that, as for sand, it is mainly in the state of transportation or deposition when the flow velocity is less than 30 cm/s. In the other words, when the flow velocity is below this critical value, the erosion of the sandcastle is minimum. According to this, the model we expect will maximize the deceleration of the sandcastle to the flow at least below the lower boundary of the erosion velocity when the sediment consumption is minimum.



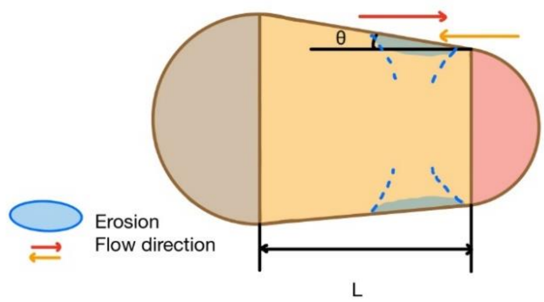


Figure 7: The Hjulström curve Figure 8: Asymmetric erosion (top view)

According to the observation, the deceleration effect of a sandcastle on water flow is positively related to the length of its transverse section L_0. However, for a certain volume of sand, longer length means lower height and smaller frontal surface area. Through qualitative analysis (Figure 8), if there is an angle between the boundary of the second part and the horizontal line, considering the flow in the opposite direction (high tide and ebb tide), the flow must show an acute angle and an obtuse angle to the boundary, respectively. So, the flow in one direction is bound to erode the sandcastle base faster, causing the base to split into two parts (which we don't want to happen).

Therefore, only when the base boundary of the second part is parallel to the horizontal line, can sandcastle best deals with the flow from different directions. Then the slowing effect of the second part of the pedestal on the flow velocity is only reflected in the extension of the longitudinal profile, and we can extend the length of first or third parts of the pedestal to achieve the same effect. Therefore, the model can be simplified into two parts under the same effect.

ISSN: 2664-9640

Under the constraint of the same volume, the best deceleration effect will be achieved when the length of the transverse section of the base is $L_0 = 2R$, where R is the circumradius of the bottom profile figure. And the relationship between the final velocity v' and the length of the transverse section L_0 is:

$$v' = (v_1 - \Delta v)^2 - \frac{L_0}{D} v_0^2$$

The relationship between the final velocity and the lower boundary of the erosion velocity will be one of the conditions to evaluate the stability of the sandcastle foundation.

The formation of capillary bridges between sand grains are the cause of the stiffness of sculptured wet sand in a sandcastle, as opposed to dry sand which can hardly or not support its own weight.[2] Qualitatively, the liquid leads to the formation of capillary bridges between the sand grains, and the curvature of the liquid interface leads to a capillary pressure causing a force of attraction between the grains.[3]

Since in many cases the humidity in the air is sufficient for liquid bridges to form between sand grains, so we can expect it to have the stiffness of an elastic body, from which the elastic modulus is introduced to measure the limit of the load-bearing capacity of the structure.

The stability of the building depends on the bottom area and its height, and the same is true for sandcastles. Therefore, we expect to obtain the relationship between the radius of the foundation and the maximum height of the sandcastle that it can support, so as to quantitatively consider the stability of a sandcastle. It found that there is an obvious difference between the observation of a small sandcastle and a sand column several meters high, and there is a distinct difference between the 20cm as the boundary (figure 9). [3] We expect to draw a conclusion that is applicable to most sandcastle lovers, so we choose the sand column with the maximum height below 20cm to study.

According to references[3], we can find the quantitative relationship between critical height h_{max} and base radius R:

$$h_{max} = \left(\frac{9J_{(-1/3)}^2}{16} \frac{GR^2}{\rho g} \right)^{1/3}$$

where G is the elastic modulus, R is the column radius, ρ is density, g is the gravitational acceleration, and J <1.8663 is the smallest positive root of the Bessel function of the first kind of order $-1/3$.[4]

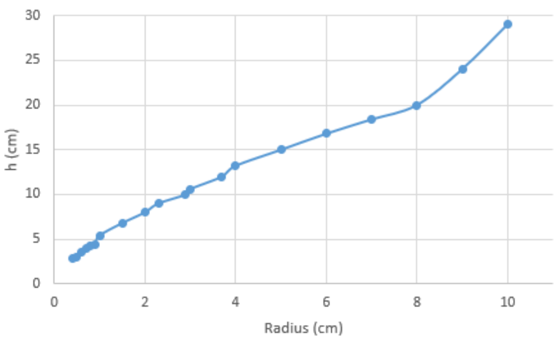


Figure 9: Relationship between the maximum critical height and base radius

We can see from figure 9 that there is a linear relationship between the maximum critical height of the sandcastle foundation and its base radius in 20cm:

$$h_{max} = 2.32R + 2.80$$

In practice, the maximum height of the sandcastle should be slightly less than the theoretical value h_{max}.

4.2.2 The Cross-sectional Shape of a Sandcastle Foundation

Considering comprehensively, we define the anti-erosion coefficient S, which can qualitatively compare the stability of the sandcastle base model:

$$S \propto C \frac{\Delta v L_0}{f D_0 h}$$

where C is a constant, D_0 is the maximum cross-sectional diameter of the sandcastle foundation, f is the frictional force.

When estimating the stability of sandcastle, we first consider making the impact force of water flow on sandcastle as small as possible. Consequently, we initially analyze the cube model, as shown in Figure 10. Next, we found that the dune structure of the cube is easy to be washed down by waves and tides, so we further considered the cylinder model, as shown in Figure 11.

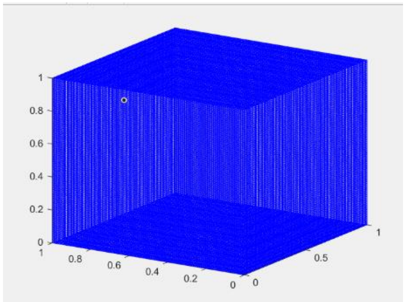


Figure 10: The cube model

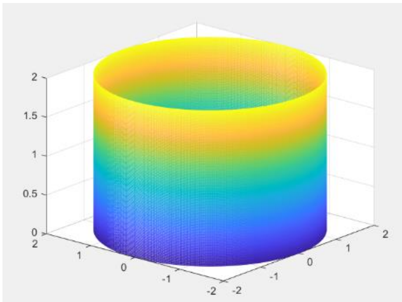


Figure 11: The cylinder model

Finally, by looking up the literature[5] , we found that the combination of multiple curved surfaces receives the least impact. The streamlined body is applied to underwater equipment design widely since it has a good static and dynamic characteristic. Based on several outline equations of the streamlined body in common used (streamlined body, semi-elliptic streamlined body, semi-ellipse with parabolic streamlined body, semi-ellipse with arc streamlined body), the drag characteristic of the underwater two-dimensional streamlined body is compared and analyzed with different structure parameters. It is shown that the semi-ellipse with arc streamlined body has the best drag characteristic.[5]

Shape \ Stability	S_1	S_2	S_3	S
Cube	0.5643	0.5001	0.6077	0.8520
Cylinder	1.0000	1.0000	1.0000	1.0000
Combination	0.9495	0.9593	1.1143	1.0400

Figure 12: Relative stability(anti-erosion) coefficients of the three different shapes

For the accuracy of the results, on the basis of consulting the literature, we further calculate each parameter affecting the stability of the three models, and the results are shown in figure 12, where S1, S2, and S3 represent the anti-erosion coefficient of section 1, 2, 3(in figure 4), respectively, and S is the anti-erosion coefficient of the three-dimensional figure (let the anti-erosion coefficient of the cylinder be 1). Because the anti-erosion coefficients of the three parts influence each other, each part with the highest anti-erosion coefficient cannot be connected singly. Only the overall anti-erosion coefficient S can reflect the overall stability of the three-dimensional graphic structure.

Therefore, combining the above improvement process, we can determine the best cross-sectional shape of the sandcastle foundation, as shown in Figure 13.

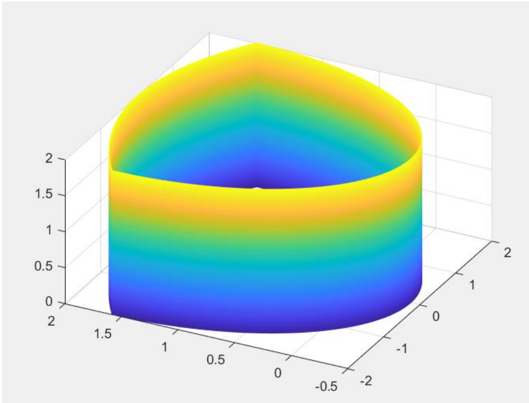


Figure 13: The best cross-sectional shape of a sandcastle foundation (the combination of the semi-ellipse surface with arc streamlined surface)

Researche[5] shows that the foundation whose inflow section is a semi-ellipse and outflow section is a streamline of an arc has the least impact force of fluid. Therefore, we let L_E and L_R denote the length of the inflow section and the length of the outflow section respectively. And we let D_0 denote the maximum cross-sectional diameter of the sandcastle foundation. Then, the curvilinear equation of the inflow section can be written as

$$y = \pm \frac{D_0}{2L_E}\sqrt{L_E^2 - x^2}$$

And, the curvilinear equation of the outflow section can be written as

$$y = \frac{D_0}{2} - \left(R - \sqrt{R^2 - x^2}\right)$$

where

$$R = \frac{4L_R^2 + D_0^2}{4D_0}$$

denotes the arc radius. And $L_E + L_R = D_0$. We expect to have a sandcastle foundation with the maximum height allowed under stable conditions to reduce the erosion of the sandcastle itself by waves and tides. Hence, $h_{max} = 20\ cm$, $L_E = 4\ cm$, $L_R = 12\ cm$, $D_0 = 16\ cm$ are taken as the reference parameter of the best sandcastle foundation model.

5. Model Two – An Optimal Sand-to-water Mixture Proportion Model

To obtain the best sand water mixture ratio of the castle foundation without using other materials (e.g. plastic or wooden supports, stones, etc.), we need to acquire further understanding about the physical and chemical relationship between the sand grains. Hence, an optimal sand-to-water mixture proportion model is discussed here.

5.1. Assumptions

The sandcastle's infrastructure is at the same distance as the water on the same beach.

The type and quantity of sand are the same.

Without using other additives or materials.

Sand-to-water mixture proportion in the whole sandcastle foundation is the same.

5.2. Establishing the Model

In order to make our sandcastle last longer on the seashore experiencing waves and tides, we need to find the best sand-to-water mixing ratio of the material through simulation and theoretical analysis.

5.2.1 Theoretical Analysis

In our daily life, it is not difficult to find that dry sand is difficult to form. When a certain amount of water is added, the adhesion and strength of the sand will be greatly improved. However, when the proportion of water added is too high, it is also difficult for the sand to maintain its geometric shape.

According to relevant literature[6], we have now quantified the effect of adding small quantities of liquid to a granular medium. Nanometre-scale layers of liquid on millimeter-scale grains dramatically increase the repose angle (the steepest stable slope that the substance can form) and allow the development of long-range correlations, or clumps.[6]

The change caused by moisture in the particle medium is mainly caused by the adhesion force of the liquid bridge between the particles. Therefore, our main task is to conduct relevant theoretical analysis, use simulation and consult the related literature to find the sand-water ratio when the adhesion is greatest.

Some related research[3] shows that: When typical values for beach sand, a roughness, below a critical liquid volume fraction about 0.2%, the bridges between the beads cannot form. At higher volume fractions, the bridge force is dominated by the curvature of the meniscus and at even higher volume fractions the bridges start to merge into larger pockets of fluid. [7] The macroscopic shear modulus G' of a macroscopic cube of dimension L containing a large number of grains can be defined as the ratio of stress and strain:

$$G' = 2(1+\mathrm{v})\frac{F_{strain}/L^2}{\Delta x/L}$$

where $\Delta x/L$ is the strain, F_{strain}/L^2 is the stress and $\mathrm{v} \approx 0.5$ is the Poisson ratio. So, we can introduce a shear module to predict the maximum strength of the sand packing.

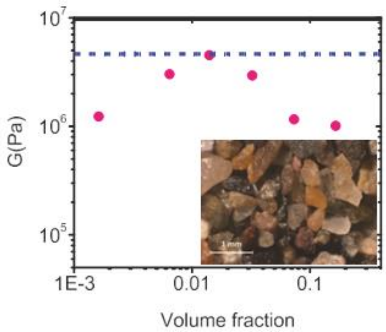


Figure 14: Scatter plot based on experimental measurements

According to the above theoretical analysis and image trend in figure 14[3], we can determine that the final optimal volume fraction of water is about 1% ~ 5%.

5.2.2 Simulation Experiment of Wave Impact

To further improve the accuracy of the model, we carried out the simulation experiment of wave impact on the sandcastle model with different water-to-sand mixture proportion under the same other conditions. Besides, to ensure the higher accuracy of the simulation experiment, we use the baffle that vibrates periodically to simulate the generation of waves in the flume (figure 15), also for simplicity, select the cylinder (figure 16) as the three-dimensional shape of the sandcastle foundation.

ISSN: 2664-9640

We put it in the same position each time, and record the corresponding sandcastle collapse time according to different water-to-sand mixture proportion (the sandcastle is regarded as collapse when the sandcastle loses 1/2 of its volume under the impact of the waves).

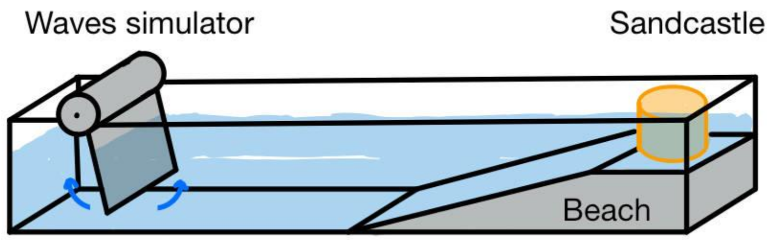


Figure 15: Simulate the impact of waves on a sandcastle

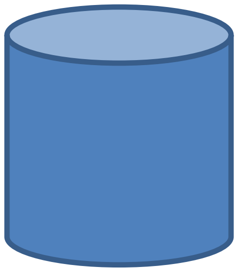


Figure 16: A schematic diagram of the shape of the sandcastle foundation

By drawing the final experimental data into a line chart (figure 17), we find that the experimental results are in good agreement with the predicted values of the theory.

Through the above theoretical analysis and simulation model, we choose 3.12% as the optimal volume fraction of water. We take the density of sand as 1.6 g/cm^3,[9] through the relationship between mass, density, and volume, we can find that 49.68 is the final sand-to-water mixture proportion for the castle foundation.

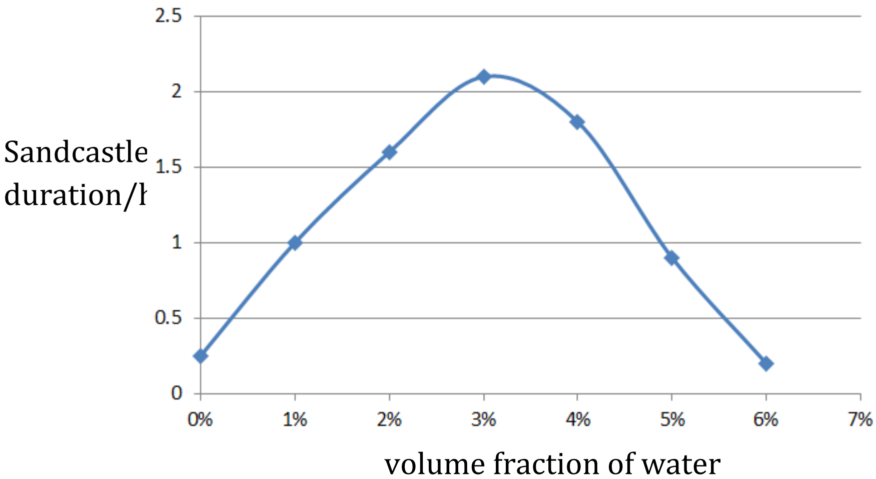


Figure 17: Relationship between duration and volume fraction of water

6. Model Extension

6.1. Rain Influence

Although people generally choose to visit the beach on a sunny day, sudden showers will cause a lot of trouble for sandcastle makers, unless they have taken into account the impact of rain and respond in advance. Therefore, we expect to improve the model in the first question so that it can maintain the maximum anti-erosion ability and stability after adding the rain factor. Since few people choose to go to the beach on rainy days, we have reason to rule out heavy rain or torrential rain, and limit the research object to showers, that is, to analyze the effects of showers on sandcastle erosion.

6.1.1 The Influence on the Sandcastle Foundation

Rainfall will increase the humidity of the sand, which in turn increases the water-sand ratio. Therefore, we establish a model to analyze this effect. The seaside area is rich in precipitation, so we might as well take Sanya in China as an example. By consulting the information on the Internet,[10] we find that the average daily precipitation in Sanya is about 5.01 mm. To facilitate the analysis, we simplify the sandcastle to a cylinder. We know the relationship between the height of sandcastle h_{max} and its bottom radius R:

$$h_{max} = 2.32R + 2.80$$

Through this equation, we can find the variation formula of water-to-sand proportion:

$$\Delta p = \frac{50.1}{2.32R + 2.80}\%$$

If we take $R = 8cm$ as an example, the change of water-to-sand proportion is about 2.35%. According to the conclusion in model two, the change of 2.35% water-to-sand proportion is fatal for sandcastles, so it is necessary to increase protective measures to improve the life of sandcastles.

6.1.2 Adjustment

The statistical data from weather bureau shows that the dense cumulus clouds about 1500 to 2000m is possible to produce intermittent rainfall which is not large, and the raindrop radius r_{rain} is about 0.5mm. The falling raindrop is affected by both gravity and air resistance, and the resistance is proportional to the speed. In the accelerated motion of the raindrop falling from the static state, the resistance changes from small to large. After a short period of time, the resistance is balanced with gravity, and finally reaches a uniform falling speed. At this time, the speed of the raindrop is called the terminal velocity v_{ter}.

According to the experimental and theoretical derivation,[8] there is a relationship between the raindrop terminal velocity and the raindrop radius (if the radius is taken mm as the unit, the velocity is taken m/s as the unit):

$$v_{ter} = 12.3r_{rain}^2 \; (r_{rain} \leq 0.5\; mm)$$

Considering the showers, the maximum terminal velocity of a raindrop is approximately 3.075 m/s.

When the raindrop falls on the sandcastle foundation at a certain speed, it will take away part of the sand particles on the slope of the base, thus causing deformation to the whole structure. The anti-erosion ability of sandcastle base to rainwater is mainly related to its slope inclination. On the one hand, the larger the slope inclination, the greater the speed of rainwater and the stronger the erosion. On the other hand, the infiltration time is short, so the effect on water-sand ratio is small.

For the sandcastle foundation located on the same beach, it can be considered that the sandcastle base with the same bottom area is scoured by the same amount of rainfall in a unit time. Rainwater can be approximately considered to fall evenly on the surface of the sandcastle

ISSN: 2664-9640

base. Therefore, for model one which we discussed above, the surface area at the bottom can be approximately regarded as the circular area (πR^2).

To make the pedestal better share the impact of rainwater, we consider a structure similar to the frustum of a cone, then its superficial surface area S_a is:

$$S_a = \pi(r^2 + RL + rL)$$

where $L = {}^{h}/_{\sin\theta}$ the length of generatrix of the frustum of a cone, R is the radius of the bottom circle, r is the radius of the top circle (figure 18). And $L^2 = (R-r)^2 + h^2$ satisfies Pythagorean theorem, so we can find the relationship between superficial area S_a and r:

$$S_a = \pi\left[r^2 + (R+r)\sqrt{(R-r)^2 + h^2}\right]$$

By substituting this formula into model one, we can obtain the best geometry parameters: $R = 8\ cm, h = 20\ cm$. We draw ${}^{1}/_{S} \sim r$ curve in figure 19, where S is the anti-erosion coefficient.

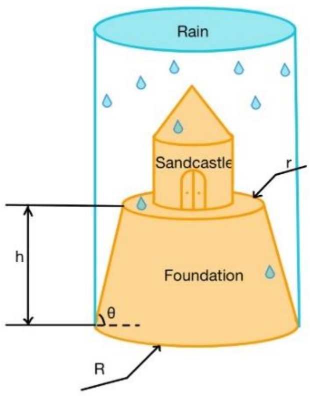


Figure 18: Improved model (considering impact of rain)

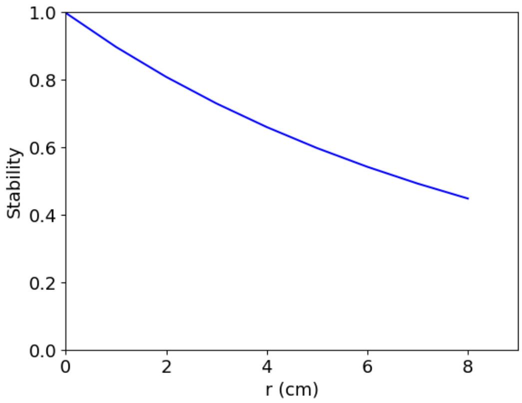


Figure 19: Let the anti-erosion coefficient the of $r = 0$ (the cone) be 1

Figure 20 shows the relationship between S and slope inclination θ (the terminal velocity of a raindrop v_{ter} is 3.075 m/s). We can see that the structure has the best corrosion resistance when $\theta \approx 74°$. In practice, the slope inclination θ should be slightly less than the theoretical value.

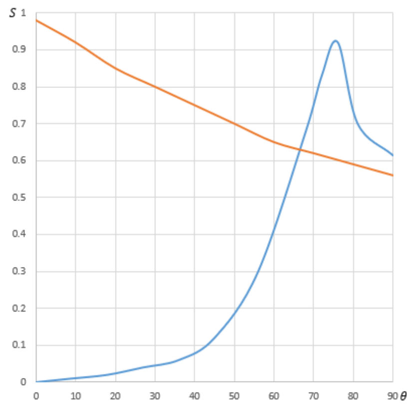


Figure 20: The best inclined angle

Considering the relationship between erosion and infiltration, we can find that the intersection coordinate corresponding to θ = 68 ° is the best inclined angle.

6.2. Other Methods

If there is no restriction on the distance between the sandcastle and the seawater, the type of sand, and the addition of other materials, and we do not consider the various repairs of the sandcastle after its erosion, we propose the following ways to prolong the service life of the sand castle.

Change the distance between the sandcastle and the seawater.

From the above analysis of model one, we get $v_1 = \sqrt{v_0^2 + 2ad} = \sqrt{1 - \frac{d}{D}} v_0$. Under the condition that the velocity of tide when rushing to the beach v_0 and the furthest distance of waves scouring the beach D are unchanged, the larger the distance between sandcastle and the edge of the sea d is, the smaller the velocity of tide when hitting the front of the sandcastle v_1 is (the ratio of d to D is less than 1 by default). From figure 7 (the critical velocity of erosion, transport, and deposition shown by the change of sediment grain size and flow velocity), we know that the erosion ability of the waves to the sandcastle will decrease significantly, so the service life of the sand castle will be prolonged.

Construct the helix on the outer surface of the foundation to drain water.

According to the above analysis of the impact of showers on model one, rainfall will increase the humidity of sand, thus increasing the sand-to-water mixture proportion for the castle foundation. To minimize the impact of rainfall, we can slightly modify the outer surface of the foundation of model one (see figure 21), to realize the drainage of the sandcastle surface. We propose two efficient, easy-to-make and beautiful drainage models: spiral canal (A) and perforated type (B), which can be easily made with tools such as shovels. Considering that punching creates holes in the sand which is more likely to collapse, so the spiral canal method of changing the appearance is more suitable.

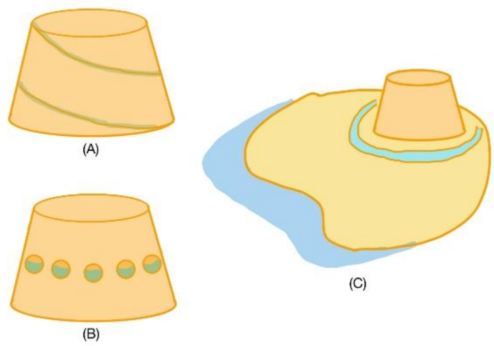


Figure 21: Other methods

Build a ‘moat’.

Another effective way to reduce waves and tides erosion is to build a moat, just like medieval castles. The moat can effectively reduce the height and velocity of the incoming waves, thus protecting the foundation. However, the too deep moat may destroy the connection between the sandcastle foundation and the beach, and then affect the stability of the foundation, so the width of the moat can make up for its lack of depth.

Based on the simulation experiment of wave impact on the sandcastle model (figure 15), we built a ‘moat’ in front of the sand castle foundation. By comparing the effects of different moat sizes on reducing the flow rate, we come to the following conclusions: At 1.25 R from the center of the approximate circle of the bottom, the moat, which is spaced equally with length a, width b and depth d at 10:6:1, is distributed on the positive impact side of the tide, which has the best effect of flow reduction.

Use the river sand.

Different types of sand have different physical properties, which in turn affects the firmness of sand castles. From a physical point of view, the cross section of sand particles used in construction must be round, because only sand particles whose cross section is approximately circular are subjected to force in all directions. Therefore, according to the properties of different sand materials,[11] we can know that river sand, which has a fine quality and is very much useful for construction purpose such as plastering and so on,[12] is more suitable for construction. Therefore, when we build the sand castle foundation, we can choose to use river sand to improve the stability of the sand bag building, and then prolong the service life of the sand castle.

7. Conclusion

In the Question one, our team compared and analyzed three models based on our preliminary ideas and initial analysis before eventually determining the best 3-dimensional geometric shape of sandcastle foundation, which is a combination of a semi-ellipse and a streamline judging by the functions below:

Inflow section:

$$y = \pm \frac{D_0}{2L_E}\sqrt{L_E^2 - x^2}$$

Outflow section:

$$y = \frac{D_0}{2} - \left(R - \sqrt{R^2 - x^2}\right)$$

Where

$$R = \frac{4L_R^2 + D_0^2}{4D_0}$$

denotes the arc radius.

According to the relationship between critical height and base radius, we calculated out a set of parameters, with $h_{max} = 20\ cm$, $L_E = 4\ cm$, $L_R = 12\ cm$ and $D_0 = 16\ cm$.

In Question two, through theoretical analysis and conducting the simulation experiment, we found that an optimal sand-to-water mixture proportion (mass) is about 49.68.

In Question three, we considered the impact of rain erosion as one of the determinants. To improve our 3-dimensional model, we comprehensively considered the erosion and infiltration of the top surface by rainfall, basing on our former research on Question one and two. Finally, we obtain the relationship between superficial area S_a and r as:

$$S_a = \pi\left[r^2 + (R + r)\sqrt{(R - r)^2 + h^2}\right]$$

And latter we calculated a reference value of θ = 68 ° as the best inclined angle, using the function above and the set relation.

In Question four, we considered other factors that can increase the stability and erosion resistance of the foundation, including the distance to the edge of the beach. Furthermore, we fine-tuned our foundation model again, changing its surface by adding a threaded drain and a moat.

So far, we have successfully built a model which is based upon experiments and analyses and in line with our expectations.

8. Strengths and Weaknesses

8.1. Strengths

Applies widely

The problem is solved by distinguishing approaches and the models employ knowledge from a variety of subjects, such as mathematics, physics etc. More importantly, model can be easily modified to accommodate to various situation.

Good flexibility

According to our analysis, our model can always find the optimum solution under various kinds of conditions.

Technical supporting

The graphs are depicted accurately and vividly, allowing the results to be understood easily. Also, we use many theory and methods to support our work. And each one of them is used reasonably and properly.

8.2. Weaknesses

Inaccuracy

When considering the value of parameters, we obtain them through a lot of different research papers which may influence the result of our model.

Simplifying assumptions

To simplify the model, we make a few assumptions which may affect the result of our model.

Lack of data

For the lack of data, we have to simulate the models abstracted from the reality to obtain the data we need which may be different from the real data.

References

[1] Hornbaker D J, Albert R, Albert I, et al. What keeps sandcastles standing?[J]. Nature, 1997, 387(6635): 765-765.
[2] Kudrolli, A. Sticky sand. Nature Mater. 7, 174–175 (2008).
[3] Pakpour, M., Habibi, M., Møller, P. & Bonn, D. How to construct the perfect sandcastle. Sci. Rep. 2, 549; DOI:10.1038/srep00549 (2012).
[4] Cox, S. J. & McCarthy, C. M. The shape of the tallest column.SIAM J. Math. Anal. 29, 547–554 (1998).
[5] Xiao Huapan, Fang Zifan, Zhu Chen,etc. Structural Drag Characteristic and Application Research of Underwater Streamlined Body[J].J of China Three Gorges Univ.（Natural Sciences),2013,35(6):92.
[6] Hornbaker, D., Albert, R., Albert, I. et al. What keeps sandcastles standing?. Nature 387, 765 (1997). https://doi.org/10.1038/42831
[7] Moller, P. & Bonn, D. The shear modulus of wet granular matter. Europhys.Lett. 80, 38002 (2007).
[8] Li Xubin, Chen Xiaolu. Physical modeling based on literacy[J/OL].http://www.qikan.com.cn
[9] Kaliakin, Victor. (2017). Example Problems Related to Compressibility and Settlement of Soils. 10.1016/B978-0-12-804491-9.00008-2.
[10] Xu Run.The average rainfall in Sanya this year is 731 mm.[N].Sanya Daily,2018-07-23(1).
[11] Q7.Introduction to the types and characteristics of building sand[EB/OL]. https:// www. qcaijing.com/industry/gongye/20190109/646.html,2019-1-9.
[12] Material Tree.Types of Sand Used for Construction[EB/OL].
[13] https://www.materialtree.com/bengaluru/blog/types-of-sand-used-in-construction/,2016-2-29.