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Research on surface fitting based on least square method

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Abstract

With the continuous development of computer and electronic technology, digital image processing technology has been popularized and applied in various fields, which makes people's requirements for image quality gradually increase. However, due to the limitations of environment and hardware conditions, the collected images often need to be fitted. Therefore, in this paper, the least square surface fitting method is studied and verified by the experimental heap algorithm. The experimental results show that the least square surface fitting algorithm has many advantages, such as accurate results, high processing efficiency and so on.

Keywords

Image processing; least square; surface fitting.

1. Introduction

In recent years, with the rapid development of digital image processing methods, more and more attention has been paid to it. Researchers have a strong interest in the related technology research. Since 1970, people began to study the surface fitting technology [1]. After years of development and research, there are a variety of surface fitting methods. In this paper, the surface fitting is divided into two ways, namely difference method and approximation method. The fitting surface obtained by interpolation method is completely through a group of data points on or near the surface. The approximation method is to use piecewise linear surface or other forms of surface to approximate this group of data. Another classification method is to classify the fitting methods according to the representation of fitting surfaces, which can be divided into the following five types: parametric surface fitting [2, 3], deformed surface fitting, piecewise linear surface fitting [4], subdivision surface fitting and implicit surface fitting. Many scholars in and out of China have also made a lot of contributions to surface fitting. Bernard et al. used the correlation point distribution model to represent the statistical shape model [5]. In Reference [6], the boundary information was used as a constraint to construct quadratic polynomial fitting surface patches, so that the surface patches have the shape recommended by pixels and their neighborhoods.

In the surface fitting calculation, the accuracy and stability of the method is the priority of researchers. The least square method can first produce the required two-dimensional surface, and optimize the surface coefficient on the basis of the two-dimensional surface. Finally, according to the specific requirements, the minimum condition principle is used to find the model that meets the requirements. This method does not need differential linearization and meets the small error and other related assumptions, so it is more widely and convenient.

To sum up, at present, the least square method for surface fitting in China is mature and stable, and the principle of least square surface fitting is not complex, which can deal with most of the surface fitting problems in the actual production and life, and is a reliable method worthy of research, The weighted least square method is used to fit the surface. The weighted least square

method is used to fit the surface in the local neighborhood, and the outlier rate is used as the weight of the data points, so as to reduce the influence of noise points and outliers on the surface and improve the local fitting effect. In the next part, this paper will introduce the least squares surface fitting and weighted least squares surface fitting, and verify it with experiments.

2. Least square surface fitting

The fitting function is established in the neighborhood of the surface data:

$$f(x) = \sum_{i=1}^{m} \alpha_i(x) p_i(x) = p^T \alpha(x) \tag{1}$$

Where$\alpha(x) = [\alpha_1(x), \alpha_2(x), \dots, \alpha_m(x)]^{\mathrm{T}}$ is the coefficient to be solved.

$$p(x) = [1, x, y, x^2, xy, y_2]^{\mathrm{T}} \tag{2}$$

$$\|x\|_{-2} = (\sum_{\mathrm{i}=1}^{n} x_i^2)^{1/2} \tag{3}$$

Equation (2) shows that the current quadratic basis function is a complete polynomial of order k.

$$\mathrm{J} = \sum_{I=1}^{n} \omega(x - x_I)[f(x) - y_I]^2 = \sum_{I=1}^{n} \omega(x - x_I)[p^T(x_I)\alpha(x) - y_I]^2 \tag{4}$$

In equation (4), $f(x)$ is the fitting function, and n is the number of points in the neighborhood, The weight at x_{I} is $\omega(x - x_I)$, $\alpha(x)$ is the undetermined coefficient. We can get the following results by derivation:

$$\frac{\partial \mathrm{J}}{\partial \alpha} = A(x)\alpha(x) - B(x)y = 0 \tag{5}$$

$$\alpha(x) = A^{-1}(x)B(x)y \tag{6}$$

The least square fitting function can be obtained by taking equation (6) into equation (1):

$$f(x) = \sum_{i=1}^{m} \Phi_i^k(x) yI = \ddot{O}^k(x)y \tag{7}$$

Where k is the order of the basis function, $\ddot{O}^k(x)$ is a shape function.

$$\ddot{O}_I^{\mathrm{Shepard}}(x) = \frac{\omega(x - x_I)}{\sum_{\mathrm{J}=1}^{n} \omega(x = x_J)} \tag{8}$$

The fitting function can be obtained by the above calculation:

$$\mathrm{K} = \frac{LN - M^2}{EG - F^2} \tag{9}$$

The Gauss curvature of the surface can be calculated according to equation (9), in which

$$\mathrm{L} = \frac{\frac{\partial \mu}{\partial x}}{\sqrt{1 + \mu^2 + \upsilon^2}} \tag{10}$$

$$M = \frac{\frac{\partial \mu}{\partial y}}{\sqrt{1 + \mu^2 + \upsilon^2}} \tag{11}$$

$$N = \frac{\frac{\partial \nu}{\partial y}}{\sqrt{1 + \mu^2 + \upsilon^2}} \tag{12}$$

$$E = 1 + \mu^2 \tag{13}$$

$$G = 1 + \nu^2 \tag{14}$$

Taking the calculated K, L, M and N into equation (9), a smooth quadric surface can be fitted by calculation.

3. Result

In this paper, Matlab platform is used to implement the specific computer program, and combined with the least square surface fitting method introduced in the previous chapter to calculate, and draw the corresponding surface, in order to verify the fitting effect of the algorithm proposed in this paper.

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In this paper, we use the equation of $\frac{\sin(x_1)}{x_1}\frac{\sin(x_2)}{x_2}$ surface as the fitting standard. The surface generated by the equation has a relatively flat area as well as a fast changing area, which is helpful to test the practicability of the algorithm.

The original surface is shown in the figure1:

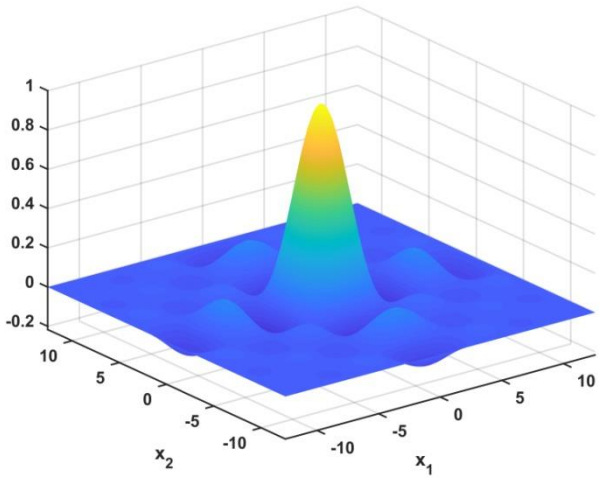

Fig. 1 Original surface

The surface obtained by the least square surface fitting method introduced in this paper is shown in figure 2:

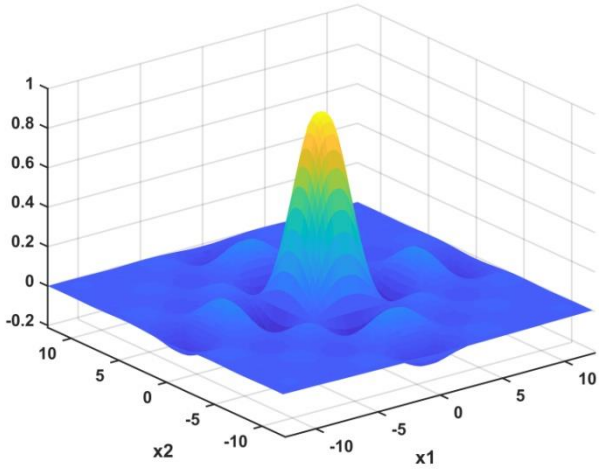

Fig. 2 Least square surface fitting results

Comparing Figure 1 and Figure 2, we can see that the least square surface fitting algorithm used in this paper can better fit the surface when the surface situation is more complex. The fitting degree of the surface is good, and the ups and downs in the original surface are well restored, and the calculation time is 1.91s, which is an efficient calculation method.

4. Summary

Surface fitting is a common problem and key technology in the fields of image processing, computer graphics and computer animation. It is a hot research topic and a difficult problem to be solved urgently in these fields. In view of this problem, this paper proposes to use the least square surface fitting method for surface fitting. Firstly, the least square surface fitting method is introduced, and then the method introduced in this paper is verified by experiments. Finally,

it is proved that the method used in this paper can effectively carry out surface fitting and is an efficient calculation method.

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