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Mathematical Modeling of Omnidirectional Mobile Robot

Wendong Huang

School of logistics engineering, Shanghai Maritime University, Shanghai 201306, China

Abstract

With the development of science and technology, the application of omnidirectional mobile robots has become more and more extensive. They are widely used in various industries such as logistics, industry, agriculture, and medical equipment. The omnidirectional mobile robot can move and rotate in any direction in the plane, which greatly improves This improves the robot's ability to work in a narrow space and improves work efficiency. In this paper, an omnidirectional mobile robot is designed. The robot realizes omnidirectional driving on level ground through the cooperation of four mecanum wheels installed on the chassis. In addition, the kinematics and dynamics models of the robot are established.

Keywords

Mobile Robot, Mecanum wheel, Omnidirectional driving, Mathematical model.

1. Overall structure description

The omnidirectional mobile robot designed in this paper mainly includes several parts such as frame, controller, power supply, motor, mecanum wheel and so on. The Mecanum wheel is a patent of Mecanum, Sweden. This omnidirectional movement is based on the principle of a center wheel with many axles located around the wheel. These angular peripheral axles convert part of the wheel's steering force into a wheel's normal force. There are already many omnidirectional mobile robots that use mecanum wheels as their actuators[1-3]. The frame is mainly constructed of aluminum alloy profiles and acrylic panels. The aluminum alloy profiles are relatively lighter, more rigid, and more resistant to oxidation than other metals. This makes the robot's frame have a higher load-bearing capacity and a higher capacity. Light body weight, and strong resistance to rust and corrosion. The moving mechanism of the robot is mainly realized by driving four mecanum wheels respectively by four DC brushless motors. The four mecanum wheels are installed on the chassis in an O-rectangular manner[4]. The controller sends control commands to each electronic controller. Each electronic speed controller decodes the data and drives the corresponding motor to rotate, thereby driving the rotation of the mecanum wheel. The controller and power supply of the omnidirectional mobile robot are encapsulated in the frame, and the four sides are wrapped by acrylic sheets. The specific structure of the omnidirectional mobile robot is shown in Figure 1.

2. Kinematics model of omnidirectional mobile robot

It can be seen from the foregoing that the mecanum wheel mechanism of the omnidirectional mobile robot is installed on the chassis in an O-rectangular layout. Now make the following assumptions:

Assumption 1: The omnidirectional mobile robot is traveling on level ground;

Assumption 2: The omnidirectional mobile robot mechanism is a rigid body.

The motion of the robot on a level ground is decomposed into three independent components: translation in the x-axis direction, translation in the y-axis direction, and rotation in the yaw-axis direction (around its geometric center), and specifies that the right direction of the car body is the positive axis direction of x, the forward direction of the vehicle body is the positive direction of the y axis, and the counterclockwise direction is the positive direction of the yaw axis. In addition, it is stipulated that the wheat wheel at the top right is wheel 1, the wheat wheel at the top left is wheel 2, the wheat wheel at the bottom left is wheel 3, and the wheat wheel at the bottom right is wheel 4. as shown in Figure 2.

Fig. 1 Structure diagram of omnidirectional robot

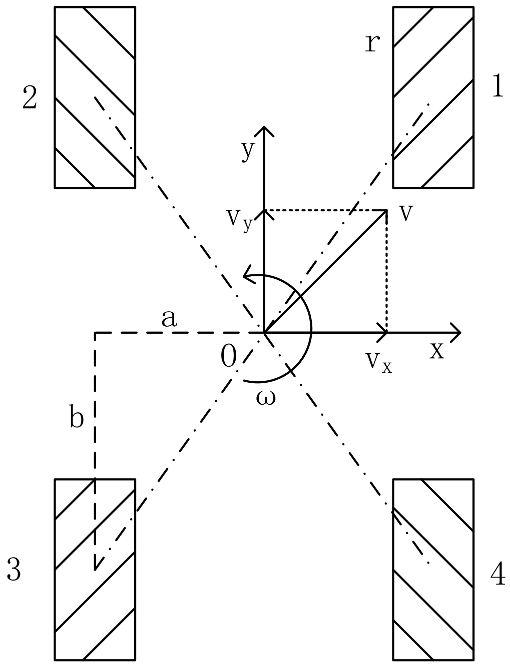

Fig. 2 Schematic diagram of mecanum wheel installation

Where,O represents the geometric center of the Mecanum wheel;

$\vec{v}$ represents the translational speed of the mecanum wheel mechanism;

$\vec{v}_x$ represents the component of the mecanum wheel mechanism in the x-axis direction;

$\vec{v}_y$ represents the component of the mecanum wheel mechanism in the y-axis direction;

$\vec{\omega}$ represents the rotation speed of the mecanum wheel mechanism around its geometric center;

a represents the distance from the axis of a single mecanum wheel to the geometric center of the mecanum wheel mechanism in the x-axis direction;

b represents the distance from the axis of a single mecanum wheel to the geometric center of the mecanum wheel mechanism in the y-axis direction;

r represents the radius of the mecanum wheel.

It is stipulated that $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ respectively represent the axis speeds of wheel 1, wheel 2, wheel 3, and wheel 4, and $\vec{\omega}_1$, $\vec{\omega}_2$, $\vec{\omega}_3$, $\vec{\omega}_4$ respectively represent the angular velocity of the wheel 1, wheel 2, wheel 3, and wheel 4, and the counterclockwise rotation direction is positive. When the omnidirectional mobile robot moves along the x-axis direction:

$$\begin{cases} \vec{v}_1 = -\vec{v}_x \\ \vec{v}_2 = +\vec{v}_x \\ \vec{v}_3 = -\vec{v}_x \\ \vec{v}_4 = +\vec{v}_x \end{cases} \tag{1}$$

When the omnidirectional mobile robot moves along the y-axis direction:

$$\begin{cases} \vec{v}_1 = \vec{v}_y \\ \vec{v}_2 = \vec{v}_y \\ \vec{v}_3 = \vec{v}_y \\ \vec{v}_4 = \vec{v}_t \end{cases} \tag{2}$$

When the omnidirectional mobile robot rotates around its geometric center:

$$\begin{cases} \vec{v}_1 = +\vec{\omega}(a+b) \\ \vec{v}_2 = -\vec{\omega}(a+b) \\ \vec{v}_3 = -\vec{\omega}(a+b) \\ \vec{v}_4 = +\vec{\omega}(a+b) \end{cases} \tag{3}$$

Since the mecanum wheel mechanism is a pure linear system, the general rotational speed relationship of the mecanum wheel mechanism in plane motion can be obtained by adding the aforementioned three motions:

$$\begin{cases} \vec{v}_1 = -\vec{v}_x + \vec{v}_y + \vec{\omega}(a+b) \\ \vec{v}_2 = +\vec{v}_x + \vec{v}_y - \vec{\omega}(a+b) \\ \vec{v}_3 = -\vec{v}_x + \vec{v}_y - \vec{\omega}(a+b) \\ \vec{v}_4 = +\vec{v}_x + \vec{v}_y + \vec{\omega}(a+b) \end{cases} \tag{4}$$

After finishing:

$$\begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix} = \begin{bmatrix} -1 & 1 & a+b \\ 1 & 1 & -(a+b) \\ -1 & 1 & -(a+b) \\ 1 & 1 & a+b \end{bmatrix} \begin{bmatrix} \vec{v}_x \\ \vec{v}_y \\ \vec{\omega} \end{bmatrix} \tag{5}$$

Let:

$$J(\alpha) = \begin{bmatrix} -1 & 1 & a+b \\ 1 & 1 & -(a+b) \\ -1 & 1 & -(a+b) \\ 1 & 1 & a+b \end{bmatrix} \tag{6}$$

That is:

$$\begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix} = J(\alpha) \begin{bmatrix} \vec{v}_x \\ \vec{v}_y \\ \vec{\omega} \end{bmatrix} \tag{7}$$

The above formula is the inverse kinematics equation of the omnidirectional mobile robot. The matrix $J(\alpha)$ represents the mapping relationship from $\left[\vec{v}_x, \vec{v}_y, \vec{\omega}\right]^T$ to $[\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4]^T$. According to the above formula, the speed of each Mecanum wheel can be solved by the speed components of the entire omnidirectional mobile robot in all directions.

If the omnidirectional mobile robot is required to travel along the positive direction of the X-axis at a constant speed m, the speed vector of the omnidirectional mobile robot is required to be $[m, 0{,}0]^T$. Through the mapping matrix $J(\alpha)$, the corresponding every The axis speed vector of each mecanum wheel is$[-m, m, -m, m]^T$, that is: when wheel 1 and wheel 3 rotate in the negative direction of the y-axis, wheel 2 and wheel 4 Rotating to the positive direction of the y-axis can realize the translation in the positive direction of the x-axis of the omnidirectional mobile robot; if the omnidirectional mobile robot is required to travel along the positive direction of the y-axis at a constant speed n, the speed vector of the omnidirectional mobile robot is required to be $[0, n, 0]^T$, through the mapping matrix $J(\alpha)$, the corresponding axis velocity vector of each mecanum wheel can be obtained as $[n, n, n, n]^T$, namely: 4 The wheels all rotate in the positive direction of the y-axis, which can realize the movement of the omnidirectional mobile robot along the positive direction of the y-axis; if the omnidirectional mobile robot is required to rotate clockwise around its geometric center at an angular velocity p, through the mapping matrix $J(\alpha)$, The corresponding axis velocity vector of each mecanum wheel can be obtained as $[p(a+b), -p(a+b), -p(a+b), p(a+b)]^T$, that is: wheel 1 and wheel 4 rotate in the positive direction of the y-axis at the speed of $p(a+b)$ respectively, wheel 2 and wheel 3 are respectively at the speed of $p(a+b)$ Turn in the negative direction of the y-axis. In other cases, and so on, as shown in Figure 3.

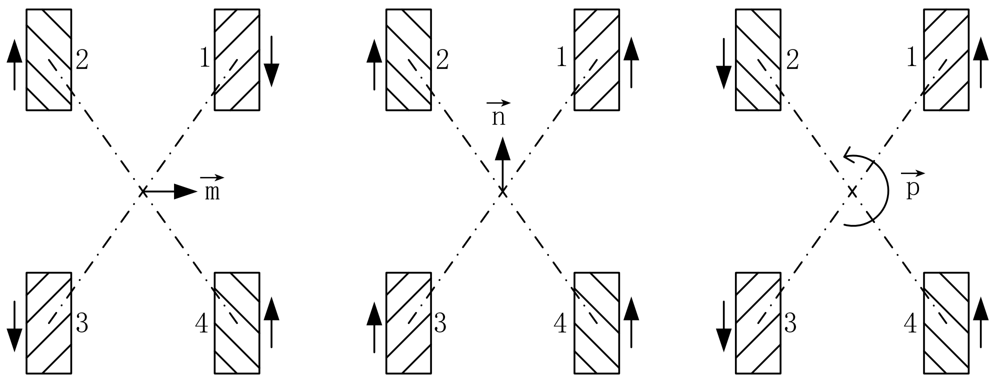

Fig. 3 Schematic diagram of the motion of the omnidirectional mobile robot

Let $J(\alpha)^+$ be the generalized inverse matrix of $J(\alpha)$, by

$$J(\alpha)^+ = \left(J(\alpha)^T J(\alpha)\right)^{-1} J(\alpha)^T \tag{8}$$

Available:

$$J(\alpha)^+ = \frac{1}{4}\begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ \frac{1}{a+b} & -\frac{1}{a+b} & -\frac{1}{a+b} & \frac{1}{a+b} \end{bmatrix} \tag{9}$$

Therefore, the kinematics equation of the omnidirectional mobile robot when driving on a level road is:

$$\begin{bmatrix} \vec{v}_x \\ \vec{v}_y \\ \vec{\omega} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ \frac{1}{a+b} & -\frac{1}{a+b} & -\frac{1}{a+b} & \frac{1}{a+b} \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix} \tag{10}$$

Convert the speed of the Mecanum wheel axis to its rotational speed, that is:

$$\begin{bmatrix} \vec{v}_x \\ \vec{v}_y \\ \vec{\omega} \end{bmatrix} = \frac{r}{4} \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ \frac{1}{a+b} & -\frac{1}{a+b} & -\frac{1}{a+b} & \frac{1}{a+b} \end{bmatrix} \begin{bmatrix} \vec{\omega}_1 \\ \vec{\omega}_2 \\ \vec{\omega}_3 \\ \vec{\omega}_4 \end{bmatrix} \tag{11}$$

3. Dynamic model of omnidirectional mobile robot

The Lagrange's equations[5] is used to model the dynamics of the omnidirectional mobile robot. In order to simplify the model and facilitate analysis, the following assumptions are made:

Assumption 1: The vehicle is running on a horizontal surface, and its potential energy remains constant at 0;

Assumption 2: Ignore the inertia of the Mecanum wheels.

It is stipulated that the speed of the omnidirectional mobile robot in the global coordinate system is $[v_x, v_y, \omega]^T$.

Kinetic energy E_k of the omnidirectional mobile robot system:

$$E_k = \frac{1}{2} m\left(v_x^2 + v_y^2\right) + \frac{1}{2} I_z \omega^2 + \frac{1}{2} I_\omega \sum_{i=1}^{4} \omega_i^2 \tag{12}$$

Where, m is the total mass of the omnidirectional mobile robot;

I_z is the moment of inertia of the omnidirectional mobile robot in the z-axis direction;

I_ω is the moment of inertia of the Mecanum wheel;

ω_i is the angular velocity of the i-th Mecanum wheel ($i = 1,2,3,4$);

Energy loss E_f during the movement of the omnidirectional mobile robot:

$$E_f = \frac{1}{2} D_M \sum_{i=1}^{4} \omega_i^2 \tag{13}$$

Where, D_M is the viscous friction coefficient of the wheel mechanism.

From the Lagrange's equations of the second type:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial F}{\partial \dot{q}} = Q \tag{14}$$

Where, q represents generalized coordinates;

$\dot{q}$ represents the generalized speed;

L represents the Lagrangian function, which is equal to the kinetic energy of the system minus the potential energy of the system. In this paper, the potential energy of the system is 0, so the Lagrangian function of the omnidirectional mobile robot is its kinetic energy;

F represents the dissipation function of the system;

Q represents the generalized force, which represents the driving torque in this paper.

Let:

$$\begin{cases} A = \dfrac{mr^2}{8} \\ B = \dfrac{I_Z r^2}{16(a+b)^2} \\ C = A + B + I_\omega \\ D = A - B \end{cases} \tag{15}$$

So:

$$M(q) = \begin{bmatrix} C & -B & D & B \\ -B & C & B & D \\ D & B & C & -B \\ B & D & -B & C \end{bmatrix} \tag{16}$$

Available:

$$M(q)\ddot{q} + D_M(q)\dot{q} = u(t) \tag{17}$$

Where, $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]^T$ represents the input of the system. By designing appropriate algorithms, the omnidirectional mobile robot can be controlled to complete functions such as speed tracking, path tracking, trajectory tracking, etc.

4. Conclusion

This paper establishes the mathematical model of kinematics and dynamics for an omnidirectional mobile robot. Through the establishment of kinematics, the mapping relationship between the overall speed of the omnidirectional mobile robot and the rotation speed of each mecanum wheel is determined; the establishment of the dynamic model of the omnidirectional mobile robot establishes the system input to the various states of the omnidirectional mobile robot The combination of the kinematics model and dynamics model of the omnidirectional mobile robot provides the basis for the control of the omnidirectional mobile robot in the future.

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