

# Preference information based multi-objective particle swarm optimization algorithm

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## Abstract

In order to improve the convergence and search efficiency of Pareto-based approaches in high dimensional space and utilize the preference requirements provided by decision makers, a multi-objective particle swarm optimization algorithm based on preference information is proposed in this paper. According to the preference functions given by decision makers, a set of reference points and reference vectors are generated in the objective space. Then the fuzzy evaluation is used to evaluate the particles according to the designed preference functions instead of Pareto dominance evaluation and the individual with the best preference degree in the particle population is regarded as the global best, which can guide the search direction and ensure the diversity. At the same time, the external archive update strategy and individual diversity control strategy are designed to guide the particle swarm to converge to the preference area of approximate front.

## Keywords

Preference information; multi-objective particle swarm optimization.

## 1. Introduction

Since the multi-objective optimization problems has frequently appeared in scientific researches and engineering applications, such as control system design, industrial scheduling, software engineering and resource allocation and so on. These practical problems often involve multiple contradictory sub-objectives, and how to find the optimal solution set is particularly important when multiple objectives are related to each other, which makes multi-objective optimization a hot research direction. Inspired by Darwin's theory of evolution, evolutionary algorithm has become an important branch of multi-objective optimization design because it does not need prior information of optimization problems and can find multiple non-inferior solutions at the same time by spatial random search. it has been widely concerned and applied in various scientific research fields. Different optimization problems will inevitably be encountered in scientific experimental research, engineering applications and people's daily life, and these optimization problems can be classified in many ways. Taking the number of objectives as the classification criterion is the most common way, which can be divided into single-objective optimization problem (SOP) and multi-objective optimization problem (MOPs). When optimizing the multi-objective problem, each optimization sub-objective will be taken into account, and these sub-objectives will restrict each other. In order to achieve the best overall solution, it is necessary to adjust the solution. Traditional methods such as weighting can be used to solve low-dimensional and simple multi-objective optimization problems, but these methods have serious deficiencies in solving large-scale high-dimensional and complex multi-objective problems. Moreover, the multi-objective optimization problem in engineering application is relatively complex, and the objective function may have some problems such as

nonlinearity, non-convexity or discontinuity, which is difficult to be solved by traditional optimization methods.

Evolutionary algorithm (Evolutionary Algorithm, EA)<sup>[1]</sup> simulates the evolution process of natural species and is an artificial intelligence computing technology based on heuristic search. The basic idea of the algorithm is inspired by Darwin's theory of evolution, which reveals the evolutionary mechanism of species in nature, and its essence is a powerful robust search and optimization process. Multi-objective evolutionary algorithm uses individual fitness information in the search process, so that those individuals with high fitness have a higher survival probability, so as to retain excellent individual evolutionary offspring. Different from the traditional mathematical optimization algorithm, the evolutionary algorithm uses the random search mechanism to search multiple individuals of the population at the same time in each evolution process, which makes the algorithm search along multiple trajectories in parallel, and improves the solving efficiency. It can also be used to solve multi-objective problems with non-linearity and discontinuity. In addition, there is no need to know the prior information of the problem in the process of evolution, so it can be used to solve the black box problem, so it is suitable for solving complex optimization problems. This has more advantages than the traditional methods developed from the field of multi-standard decision-making. The first evolutionary algorithm for solving multi-objective optimization problems can be traced back to the 1980s, that is, the genetic algorithm based on vector evaluation (Vector-evaluated Genetic Algorithm, referred to as VEGA), first proposed by Schaffer in 1985 is regarded as the pioneering work of multi-objective evolutionary algorithm<sup>[2]</sup>. Since the 1990s, more and more scholars and professionals have devoted themselves to the research of multi-objective evolutionary algorithms. Although these algorithms use different methods and strategies to improve optimization performance, most of them use a basic framework to run. It has become a significant difficulty to balance the ability between the local search and global search in multi-objective evolutionary algorithm<sup>[3]</sup>.

## 2. Multi-objective Optimization Standard Test Set

In order to facilitate the comparison of the optimization performance of different algorithms, many different types of standard test sets have been proposed. These standard test sets design their own attribute characteristics according to the different characteristics of multi-objective optimization problems in real life, focusing on the difficulty of optimization in different aspects. This article mainly introduces several widely used standard test sets as test functions, as described below<sup>[4]</sup>.

WFG<sup>[5]</sup> was built by Huband et al in 2006 using a flexible toolkit. Similar to the DTLZ test set, the dimensions of WFG decision variables and the number of optimization objectives can be arbitrarily expanded. WFG also contains 9 test functions, among which WFG1-WFG3 is a multi-objective optimization test set with regular shape real Pareto front, while the shape of real Pareto front of WFG4-WFG9 test set is irregular.

MaF<sup>[6]</sup> is a test set proposed by Cheng et al in the 2017 IEEE Congress on Evolutionary Computation High-dimensional Multi-objective Optimization Competition. MaF contains a total of 15 test functions, mainly for high-dimensional multi-objective optimization problems with more than 3 targets, and many optimization problems with irregular real Pareto frontiers. Among them, the real Pareto frontier of MaF2 is incoherent, the real Pareto frontier of MaF4 is inverted, and the real Pareto frontier of MaF13 is degenerate.

## 3. Multi-objective Optimization Performance Index

This section introduces two commonly used comprehensive measures of convergence and diversity, inverse generation distance (IGD) and hypervolume (HV). And this paper uses these

two indicators to measure the performance of the algorithm. The definition of IGD<sup>[7]</sup> is as follows:

$$IGD(X, Y) = \frac{\sum_{y \in Y} \min_{x \in X} dis(y, x)}{|Y|} \quad (1)$$

Where X is the objective function value of a series of non-dominant solutions obtained by running a MOEA, and Y is the reference point, dis (y to x) uniformly sampled on the front surface of the real Pareto, which is the Euler distance between points y and x. We get IGD by calculating the average shortest distance between each point in Y and the midpoint in X. This index shows the convergence and diversity of the understanding set. The greater the IGD index value, the worse the convergence and diversity of the solution set.

The definition of HV<sup>[8]</sup> is as follows:

$$HV(f^{ref}, X) = \Lambda \left( \bigcup_{X_n \in X} [f_1(X_n), f_1^{ref}] \times \dots \times [f_m(X_n), f_m^{ref}] \right) \quad (2)$$

Where represents the size of the covering space of the solution set X, while is the reference point, denotes Lebesgue measure. The index indicates the convergence and diversity of the understanding set. The smaller the HV index, the worse the convergence and diversity of the solution set.

#### 4. Introduction of Preference Information

Researchers have proposed many ways of introducing preferences, which can effectively solve a series of different problems. According to different decision-making methods, the preference multi-objective optimization algorithm is divided into the following three categories:

the first decision-making method; the first decision-making method is to add the preference information of the decision maker before the algorithm starts to run. This method is simple, intuitive and efficient, but at the beginning, it is difficult for decision makers to specify their own preference information accurately, so that the final optimal solution is not necessarily the solution that decision makers are most interested in.

interactive decision-making method; interactive decision-making method is to make a decision while searching while the algorithm is running. However, when the algorithm does not find the solution that the decision maker is interested in, the decision maker can not make the correct decision, and when to interrupt the search decision is also a problem. Interrupting too early will affect the performance of the algorithm. If interrupted too late, some optimal solutions will be lost.

Post-decision-making method. Make a decision after the end of the algorithm. After the optimization algorithm finally obtains the Pareto optimal solution, it is only necessary to select the satisfactory solution from these Pareto optimal solutions.

This part mainly introduces the components of MOPSO: the selection strategy of global optimal solution and individual optimal solution, the maintenance strategy of external archive set and the parameter setting strategy of PSO. Among them, the individual optimal solution and the global optimal solution jointly control the motion direction of a particle. Among them, the selection of the global optimal solution mainly focuses on the non-dominant solution which is relatively sparse in the external file set as the global optimal solution, in order to avoid the whole population converging to a single region and increase diversity. The individual optimal solution usually chooses the dominant solution by comparing the dominant relationship between the current solution and the historical optimal solution. Using a certain strategy to select individual optimal solution and global optimal solution in pairs to balance development and mining can better enhance the convergence and diversity. The Pareto optimal solution set is maintained and stored in an external archive set. During the operation of the algorithm, the

solution stored in the external archive set can be selected as the global optimal solution, thus guiding the whole population to move to a more promising place. After the end of the algorithm operation, the external archive set outputs the Pareto front end of the algorithm, taking into account convergence and diversity as far as possible. PSO's unique parameter setting strategy can balance the development and exploitation of the population in the process of evolution. Flexible setting of parameters can accelerate the convergence speed and improve the diversity of the algorithm.

The researchers put forward the method based on weight<sup>[9]</sup>, citation point, reference direction, G domination, R domination, bipolar preference, beam search, angle information and so on.

### 4.1. G-dominance algorithm

Since Pareto domination was proposed by Pareto, it has been deeply recognized by the majority of researchers. Pareto dominance relation is also widely used in multi-objective optimization algorithms. Because the Pareto dominance relation plays a screening role in population selection, we can also enhance the Pareto dominance relation to add the preference information of decision makers. The g-dominance method is a preference introduction method by strengthening the Pareto domination relation. It improves the Pareto domination relation on the basis of the reference point and increases the selection pressure. It not only has the advantage of the reference point method, but also can quickly find the preference solution. This method does not need to use the ASF function to obtain some preference solutions near the reference point. The principle of the g-dominance method is described in detail below.

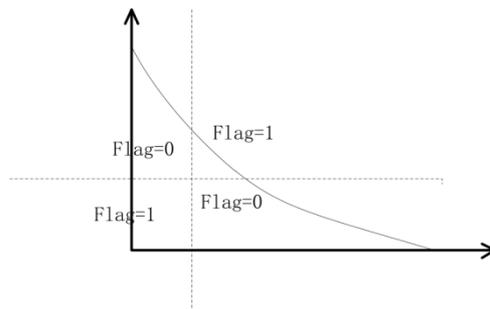


Fig 1. G-dominance Flag diagram with two targets

First of all, a preference point is given by the decision maker, which consists of a relationship between the preference point and the individuals in the population. The definition of this relationship is shown in the formula (3). We call this relationship as. In the case of two targets, the whole feasible region is divided into four parts by this reference point, as shown in figure 3 (the intersection of the dotted lines in the figure is the preference point. Then define the dominant relationship as shown below:

$$Flag_p(g) = \begin{cases} 1, & \text{if } p_i \leq g_i, \forall i = 1, \dots, M \\ 1, & \text{if } p_i \geq g_i, \forall i = 1, \dots, M \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

G-dominance definition: any two individuals X and Y in the target space are called X g-dominance Y if they satisfy one of the two conditions of the lower plane.

(1)  $Flag_p(X) > Flag_p(Y)$

(2)  $Flag_p(X) = Flag_p(Y)$ , And there is  $X_i \leq Y_i, \forall i = 1, 2, \dots, M$  exists and at least j satisfies  $X_j < Y_j$ ;

In the g-dominance method, from the definition as , we can see that this method is highly dependent on the reference point, which has both advantages and disadvantages. The preferred solution can be obtained by setting the reference point, and the decision maker can set a reference point close to the preference area. It is relatively simple and intuitive for decision

makers. However, when the reference point is set in some special areas, the preference solution that meets the expectations of decision makers can not be obtained or even the optimal solution. The g-dominance method uses an interactive decision-making process, which interacts with the reference point set by the decision maker throughout the process, but there is no feedback to the decision maker.

determination of preference areas

The preference information provided by the decision maker can be integrated through the various methods described in the previous section, or it can be determined by the acceptable error range. If the optimal solution is within the error range, the optimal solution can explain the preference solution expected by the decision maker. Koksalan and Karalan<sup>[10]</sup> represent this error by preset a set of target ranges, but what should be the scope of the preference region? How do we measure boundaries?

How to determine the preference region of the decision maker in the preference multi-objective optimization algorithm is related to how many non-dominant solutions the decision maker chooses as the final decision. If the decision area is too large, then there is no point in setting preferences. If the decision area is too large, the final preference solution set is no different from the original Pareto surface. Did not achieve the original intention of setting preferences. If the preference region setting is too small, there will be a great pressure on the selection of the algorithm, and it can not accurately meet the preference requirements of decision makers, because the preference of decision makers is only a range and will not be very accurate. If it is particularly accurate, there is no need to do any optimization, decision makers can make decisions directly, there is no need for preference optimization algorithms at all.

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## 5. Summary

This paper introduces a multi-objective optimization algorithm with preference information. Firstly, the introduction method of preference information is introduced, and the principles and types of several preference introduction methods are introduced in detail. The introduction method of preference information has a great influence on the complexity and convergence speed of the algorithm. Therefore, the effect of designing an excellent method of introducing preference information on the performance of preference multi-objective optimization algorithm can not be ignored. Then it explains how to determine the scope of the preference region to ensure that the preference region is controllable. The determination of the preference region range is an important factor that determines the feasibility of the preference multi-objective optimization algorithm. I put forward my own opinions on the determination of the preference area. Finally, several commonly used distribution maintenance methods are introduced, and the preference for the distribution of solutions is very important to decision-makers and can influence decision-makers to make decisions. Therefore, we should focus on the maintenance of the distribution of the preferred solution. Combining preference information with PSO is the development trend of particle swarm optimization in the future.

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