ISSN: 2664-9640

Tracking Control for Nonholonomic Mobile Robots

Qi Luo, Changzhong Chen, Xin Liu and Zengcheng Sun

College of Automation and Information Engineering, Sichuan University of Science & Engineering, Zigong, Sichuan 643000, China.

Abstract

In the case of non-coincidence in the center of the centroid and drive wheel axis, the trajectory tracking control problem of wheeled mobile robots is studied based on the kinematics model. Firstly, based on the kinematic model, a kinematics controller is designed to implement tracking of the ideal trajectory. Finally, the validity of the design controller is verified by MATLAB simulation experiment.

Keywords

Mobile robot, Kinematics model, Lyapunov function.

1. Introduction

Recently, in many areas such as industrial, civil, national defense, logistics, medicine, etc. The wheel mobile robot (WMR) has been widely used due to the advantages of light weight, large load capacity, reliable, flexible, stable control, simple, etc. But the wheeled mobile robot is an uncertain nonlinear system [1]. When the WMR constrains the wheel's "pure rolling without slipping," it is also a typical kind of nonholonomic systems characterized by kinematic constraints [2]. Because there is a non-intact constraint that makes its tracking control problem with great challenging. In the past few decades, there has been tremendous research on the nonholonomic wheeled mobile robot.

The trajectory tracking problem is one of the most popular problems on the WMR [3-5]. Y. Kanayama et al. proposed a stable tracking control law using Lyapunov direct method, and mentioned the uniformly asymptotically stable concept using Lyapunov's linearization method [6]. Track tracking control is designed under extreme coordinate system in [7]. The associated item of position error and direction angle error is introduced into the Lyapunov function and the controller is designed by this function in [8]. The system is linearized by Walsh [9] near the vicinity of the expected trajectory. SO, linear time-variable system control law is designed by him. And, there by achieving part tracking of the original system. The Backstepping method is used by Fierro R [10] adaptive tracking control of non-intact mobile robots, this controller design is simpler. To further improve the trajectory tracking control performance of wheeled mobile robots. Some scholars use nonlinear systems to control the control of wheeled mobile robots. A typical example is state feedback linearization control [11]. Rossomando F G [12] designed trajectory tracking control through a fuzzy system and neural network, the controller implements position tracking but ignored the attitude tracking. Variable structure control, fuzzy control, adaptive control is proposed in [13-15], these control laws are obtained by extension of [16]. In addition, In addition, sliding mode control can also design mobile robots controllers [17]. Controller can be designed by fuzzy neural network in [18-20]. There is an interference problem in three rounds of full-to-directional mobile robot trajectory, and the passive self-antisense controller is designed in [21].

Based on the previous results, this paper addresses the trajectory tracking problem for the WMR with parameter uncertainties. Firstly, based on the kinematic model, a new Lyapunov function is proposed [22]. Secondly, a kinematics controller is designed by using the function

ISSN: 2664-9640

to guarantee that the tracking error. Finally, effectiveness of the controller is verified by simulation.

2. Problem Statements

A typical example of a nonholonomic mobile robot is shown in Fig.1. The two front wheels of the robot are controlled independently by motors. By adjusting the respective input voltages of the drive wheel to reach the speed difference of the two front wheels, thereby realizing changes in the position of the mobile robot, thereby performing trajectory tracking control. The two rear wheels of the mobile robot are followed by the wheels, because there is no motor, so it only serves the role of the robot without changing the direction.

The movement of the wheel mobile robot is pure rolling without slipping. This rolling constraint is manifested to limit the instantaneous speed of the wheel and the ground contact point, that is, the forward direction of the wheel mobile robot is always perpendicular to the axle, so therefore the robot is subject to the following non-complete constraints:

$$\dot{x}\sin\theta - \dot{y}\cos\theta + d\dot{\theta} = 0 \tag{1}$$

The positional state of the wheel mobile robot is indicated by the position of its centroid P in the coordinate system and the navigation angle θ.

where $(x \quad y)$ denotes the position P of the center of mass, d is the distance between the geometric center point and the mass center point of the robot, further $d > 0$. The actual position of the wheeled mobile robot is the coordinates for $P = [x_c \quad y_c \quad \theta_c]^T$. Control law $q = [\nu \quad \omega]^T$. They are control input in motion models and ν is the forward velocity while ω is the angular velocity of the robot. By the formula (1), the kinematics of the robot can be modeled by the following differential equations:

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{bmatrix} = \begin{bmatrix} \cos\theta_c & -d\sin\theta_c \\ \sin\theta_c & \cos\theta_c \\ 0 & 1 \end{bmatrix} q \tag{2}$$

where $q = [\nu \quad \omega]^T$.

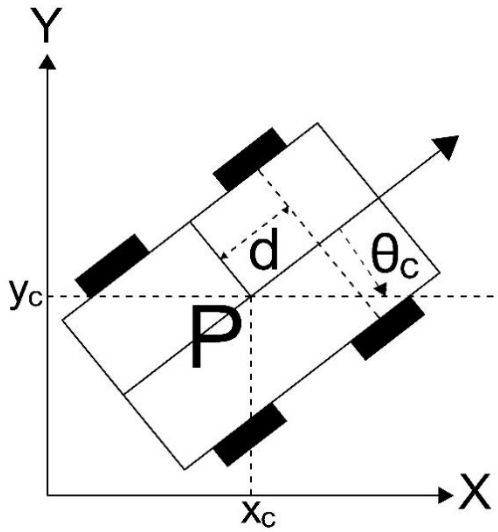


Figure 1: Model of wheeled mobile robot

In this control system, two postures are used: the reference posture $P_r = [x_r \quad y_r \quad \theta_r]^T$, and the current posture $P_c = [x_c \quad y_c \quad \theta_c]^T$. We formulate the mobile robots with error state equations. Figure2. represents the concept of error posture.

ISSN: 2664-9640

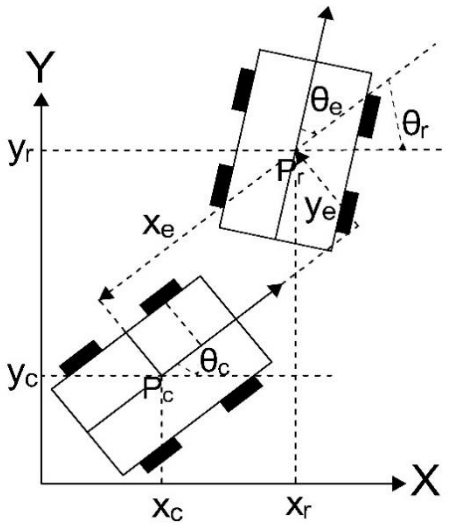


Figure 2: The sketch map of posture error of mobile robot

This is the difference between P_r and P_c:

$$P_e=\begin{bmatrix}x_e\\y_e\\\theta_e\end{bmatrix}=\begin{bmatrix}\cos\theta_c & \sin\theta_c & 0\\-\sin\theta_c & \cos\theta_c & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r-x_c\\y_r-y_c\\\theta_r-\theta_c\end{bmatrix} \tag{3}$$

where P_e denotes track tracking error of mobile robots.

We can conclude that Position error dynamic equation (4):

$$\dot{p_e}=\begin{cases}\dot{x_e}=y_e\omega-\nu+\nu_r\cos\theta_e-d\omega_r\sin\theta_e\\\dot{y_e}=-x_e\omega-d\omega+d\omega_r\cos\theta_e+\nu_r\sin\theta_e\\\dot{\theta_e}=\omega_r-\omega\end{cases} \tag{4}$$

Proof. Using Equation (1), (2) and taking the time derivative of (3)

$$\begin{aligned}\dot{x_e}&=(\dot{x_r}-\dot{x_c})\cos\theta_c+(\dot{y_r}-\dot{y_c})\sin\theta_c+\left[\dot{\theta_c}(x_r-x_c)\sin\theta_c+\dot{\theta_c}(y_r-y_c)\cos\theta_c\right]\\&=\dot{x_r}\cos\theta_c-\dot{x_c}\cos\theta_c+\dot{y_r}\sin\theta_c-\dot{y_c}\sin\theta_c+y_e\omega\\&=y_e\omega+\dot{x_r}\cos\theta_c+\dot{y_r}\sin\theta_c-(\nu\cos\theta_c-\omega d\sin\theta_c)\cos\theta_c-\sin\theta_c\,(\nu\sin\theta_c+\\&\quad\omega d\cos\theta_c)\\&=y_e\omega+\dot{x_r}\cos\theta_c+\dot{y_r}\sin\theta_c-\nu\\&=y_e\omega-\nu+\dot{x_r}\cos(\theta_r-\theta_e)+\dot{y_r}\sin(\theta_r-\theta_e)\\&=y_e\omega-\nu+\dot{x_r}(\cos\theta_r\cos\theta_e+\sin\theta_r\sin\theta_e)+\dot{y_r}(\sin\theta_r\cos\theta_e-\cos\theta_r\sin\theta_e)\\&=y_e\omega-\nu+(\dot{x_r}\cos\theta_r+\dot{y_r}\sin\theta_r)\cos\theta_e+(\dot{x_r}\sin\theta_r-\dot{y_r}\cos\theta_r)\sin\theta_e\\&=y_e\omega-\nu+\nu_r\cos\theta_e-d\omega_r\sin\theta_e\end{aligned}$$

$$\begin{aligned}\dot{y_e}&=-\sin\theta_c\,(\dot{x_r}-\dot{x_c})+\cos\theta_c\,(\dot{y_r}-\dot{y_c})-\dot{\theta_c}\cos\theta_c\,(x_r-x_c)-\dot{\theta_c}\sin\theta_c\\&\quad(y_r-y_c)\\&=-x_e\omega-\dot{x_r}\sin\theta_c+\dot{x_c}\sin\theta_c+\dot{y_r}\cos\theta_c-\dot{y_c}\cos\theta_c\\&=-x_e\omega-d\omega-\dot{x_r}\sin\theta_c+\dot{y_r}\cos\theta_c\\&=-x_e\omega-d\omega-\dot{x_r}\sin(\theta_r-\theta_e)+\dot{y_r}\cos(\theta_r-\theta_e)\\&=-x_e\omega-d\omega-\dot{x_r}(\sin\theta_r\cos\theta_e-\cos\theta_r\sin\theta_e)+\dot{y_r}(\cos\theta_r\cos\theta_e+\sin\theta_r\\&\quad\sin\theta_e)\\&=-x_e\omega-d\omega-(\dot{x_r}\sin\theta_r-\dot{y_r}\cos\theta_r)\cos\theta_e+(\dot{x_r}\cos\theta_r+\dot{y_r}\sin\theta_r)\sin\theta_e\\&=-x_e\omega-d\omega+d\omega_r\cos\theta_e+\nu_r\sin\theta_e\end{aligned}$$

$$\dot{\theta_e}=\dot{\theta_r}-\dot{\theta_c}=\omega_r-\omega$$

ISSN: 2664-9640

The trajectory tracking control problem studied in this paper can be converted to design the corresponding speed controller ν and ω, and the wheel mobile robot is at any initial value, and the tracking error can get better convergence. In other words, $\lim_{t\to\infty}\|[x_e \quad y_e \quad \theta_e]^T\| = 0$.

3. Design of controller

The kinematic tracking controller for system (4) is designed:

$$\begin{cases} \nu = \nu_r \cos\theta_e - 2k_3\omega\sin\frac{\theta_e}{2} + k_1\left(x_e + 2d\sin^2\frac{\theta_e}{2}\right) \\ \omega = \omega_r + \nu_r\left[\frac{1}{k_2}\left(y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}\right)\cos\frac{\theta_e}{2} + \frac{1}{k_3}\sin\frac{\theta_e}{2}\right] \end{cases} \tag{5}$$

where $k_1 > 0, k_2 > 0, k_3 > 0, \nu_r > 0$.

Theorem 1 The control inputs given in (5) can guarantee the asymptotic convergence of the states of the closed loop system defined by (4) in the sense that.

Proof. To prove Theorem 1, we consider a Lyapunov function candidate

$$V = V_1 + V_2 + V_3 \tag{6}$$

where $V_1 = \frac{1}{2}\left(x_e + 2d\sin^2\frac{\theta_e}{2}\right)^2$, $V_2 = \frac{1}{2}\left(y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}\right)^2$

and $V_3 = 4k_2\sin^2\frac{\theta_e}{4}$

Using Equation (4), (5) and taking the time derivative of (6) we can conclude that

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \tag{7}$$

$$\begin{aligned} \dot{V}_1 &= \left(x_e + 2d\sin^2\frac{\theta_e}{2}\right)\left(\dot{x}_e + d\dot{\theta}_e\sin\theta_e\right) \\ &= A\left(y_e\omega + 2k_3\omega\sin\frac{\theta_e}{2} - k_1A - d\omega\sin\theta_e\right) \\ &= -k_1A^2 + A\omega\left(y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}\right) \\ &= -k_1A^2 + AB\omega \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{V}_2 &= \left(y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}\right)\left(\dot{y}_e - d\dot{\theta}_e\cos\theta_e + k_3\dot{\theta}_e\cos\frac{\theta_e}{2}\right) \\ &= B\left(-x_e\omega - d\omega + d\omega\cos\theta_e + \nu_r\sin\theta_e + k_3\omega_r\cos\frac{\theta_e}{2} + k_3\omega\cos\frac{\theta_e}{2}\right) \\ &= B\omega(-x_e - d + d\cos\theta_e) - k_3\cos\frac{\theta_e}{2}B\left(\frac{\nu_r}{k_2}\cos\frac{\theta_e}{2}B + \frac{\nu_r}{k_3}\sin\frac{\theta_e}{2} + \omega_r\right) \\ &\quad +\nu_r\sin\theta_e B + +k_3\omega\cos\frac{\theta_e}{2}B \\ &= B\omega\left(-x_e - d + d\left(1 - 2\sin^2\frac{\theta_e}{2}\right)\right) - k_3\frac{\nu_r}{k_2}\cos^2\frac{\theta_e}{2}B^2 + \frac{1}{2}\nu_r\sin\theta_e B \\ &= -AB\omega + \frac{1}{2}\nu_r\sin\theta_e B - k_3\frac{\nu_r}{k_2}\cos^2\frac{\theta_e}{2}B^2 \end{aligned} \tag{9}$$

$$\dot{V}_3 = k_2\dot{\theta}_e\sin\frac{\theta_e}{2} = k_2(\omega_r - \omega)\sin\frac{\theta_e}{2}$$

$$= k_2\omega_r \sin\frac{\theta_e}{2} - k_2 \sin\frac{\theta_e}{2}\left(\frac{\nu_r}{k_2}\cos\frac{\theta_e}{2}B + \frac{\nu_r}{k_3}\sin\frac{\theta_e}{2} + \omega_r\right)$$

$$= -\frac{1}{2}\nu_r \sin\theta_e B - k_2\frac{\nu_r}{k_3}\sin^2\frac{\theta_e}{2} \tag{10}$$

$$\dot{V} = -k_1 A^2 - k_3\frac{\nu_r}{k_2}\cos^2\frac{\theta_e}{2}B^2 - -k_2\frac{\nu_r}{k_3}\sin^2\frac{\theta_e}{2} \le 0 \tag{11}$$

where $A = x_e + 2d\sin^2\frac{\theta_e}{2}$, $B = y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}$

Only $x_e = y_e = \sin\frac{\theta_e}{2} = 0$, Formula (11) is equal to zero.

Definitely, $V > 0$, consequently function V is positive definite. The time derivative of V is negative definite from Inequality (11). By using the Lyapunov Lemma, we could have

$$\lim_{t\to\infty} A = \lim_{t\to\infty}\left(x_e + 2d\sin^2\frac{\theta_e}{2}\right) = 0 \tag{12}$$

$$\lim_{t\to\infty} B = \lim_{t\to\infty}\left(y_e - d\sin\theta_e + 2k_3\sin\frac{\theta_e}{2}\right) = 0 \tag{13}$$

$$\lim_{t\to\infty}\sin\frac{\theta_e}{2} = 0 \tag{14}$$

Using Equation (14), we could have

$$\lim_{t\to\infty}\theta_e = 2k\pi \Leftrightarrow \lim_{t\to\infty}\theta_e = 0 \tag{15}$$

Then,

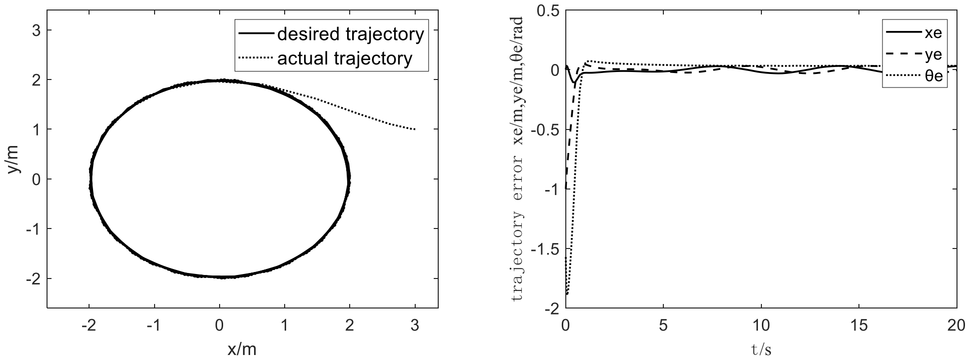
$$\begin{cases}\lim\limits_{t\to\infty} x_e = 0\\ \lim\limits_{t\to\infty} y_e = 0\end{cases} \tag{16}$$

For the controller, the stability is proved using Lyapunov stability theory.

4. Simulation

In this section, a simulation will be provided to show the effectiveness of the proposed control strategy. We present an example to illustrate the effectiveness of the proposed design scheme. The simulation is implemented and its result is shown in Figure 3 and Figure 4, respectively.

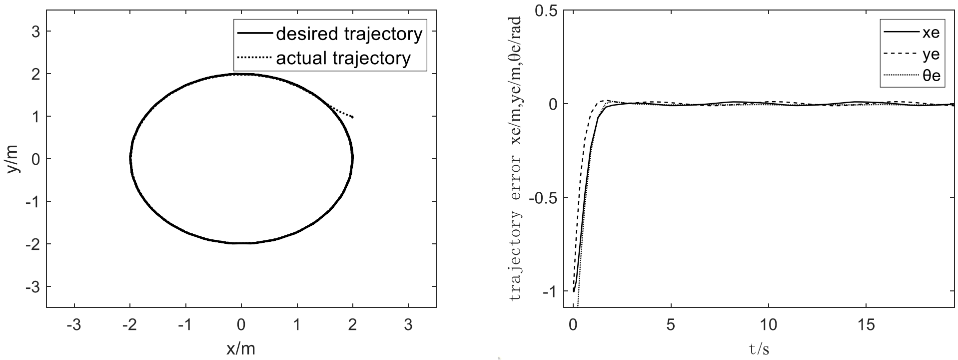
From Figures 3, the WMR's parameters in (2) are given as $d = 0$ m,in the simulation, the parameters of the controller are chosen as $k_1 = 2$, $k_2 = 0.5$, $k_3 = 1$.The initial position and orientation of the reference input $P = [3 \quad 1 \quad \pi]^T$.



Tracking track of circular expected (b) Convergence curve of x_e, y_e and θ_e

Figure 3: The results of kinematic tracking (d=0 m)

From Figures 2, the WMR's parameters in (2) are given as $d = 0.1$m,in the simulation, the parameters of the controller are chosen as $k_1 = 0.7$, $k_2 = 0.05$, $k_3 = 1$.The initial position and orientation of the reference input $P = [2 \quad 1 \quad \pi]^T$.



Tracking track of circular expected (b) Convergence curve of x_e, y_e and θ_e

Figure 4: The results of kinematic tracking (d=0.1m)

As shown in Figure 4, the tracking error are tend to zero as well. The effectiveness of the proposed controller is verified by the simulations.

5. Conclusion

In this paper, based on the kinematic model, a Lyapunov function is proposed.And a kinematics controller is designed by using the function to guarantee that the tracking error. Through Figures 3 and 4, we found that the parameter D affects the trajectory tracking of the mobile robot. However, the trajectory can be tracked, the error converge is not very good. In the future, we will consider the influence of disturbances on the system.

References

[1] Tzafestas, Spyros G. Mobile Robot Control and Navigation: A Global Overview[J]. Journal of Intelligent & Robotic Systems, 2018, 91: 35-58.

[2] Peng S Y, Shu W X. Adaptive Fuzzy Integral Terminal Sliding Mode Control of A Nonholonomic Wheeled Mobile Robot[J]. Mathematical Problems in Engineering Theory Methods & Applications, 2017,4:1-12.

[3] Binh N T, Tung N A, Nam D P, et al: An Adaptive Backstepping Trajectory Tracking Control of A Tractor Trailer Wheeled Mobile Robot[J]. International Journal of Control, Automation & Systems, 2019, 17(2): 465-473

[4] Nascimento T P, Carlos Eduardo Trabuco Dórea, Gonalves L M G: Nonlinear Model Predictive Controlfor Trajectory Tracking of Nonholonomic Mobile Robots: A Modified Approach[J]. International Journal of Advanced Robotic Systems, 2018: 1-14.

[5] Koubaa Y, Boukattaya M, Damak T:Adaptive Sliding Mode Control for Trajectory Tracking of Nonholonomic Mobile Robot with Uncertain Kinematics and Dynamics[J]. Applied Artificial Intelligence, 2018, 32(7-10): 924-938.

[6] Kanayama Y, Kimura Y, Miyazaki F, et al: A Stable Tracking Control Method for An Auto-n omous Mobile Robot[C]. Proceedings of IEEE International Conferen- ce on Roboticsa-nd Automation. Cininnati USA: IEEE, 1990: 384-389.

[7] Park B S, Yoo S J, Park J B, et al: Adaptive neural sliding mode control of nonholonomic wheeled mobile robots with model uncertainty[J]. IEEE Trans on Control Systems Technology, 2009, 17(1): 207-214.

[8] Kim D H, Oh J H: Globally Asymptotically Stable Tracking Control of Mobile Robots[C]. IEEE International Conference on Control Applications. Trieste: IEEE, 1998, 1297-1301.

[9] Wals h G, Tilbury D, Sastry S et al: Stabilization of trajectories for systems with nonholonomic constrain ts. IEEE Trans. Automatic Control, 1994, 39(1): 216～ 222

ISSN: 2664-9640

[10] Fierro R, Lewis F L: Control of a nonholonomic mobile robot :backstepping kinematics into dynamics[A] .IEEE proceedings of 34th conference on decision and control [C] , New Orleans, LA, December, 1995 :3805-3810.

[11] KIM D H: Tracking control of a two-wheeled mobile robot using input-output linearization [J]. Control Engineering Practice, 1999, 7(3): 369 – 373.

[12] Rossomando F G, Soria C, Carelli R: Neural networkbased compensation control of mobile robots with partially known structure[J]. IET Control Theory & Applications, 2012, 16(2): 1851-1860.

[13] Fukao T, Nakagawa H, Adachi N: Adaptive tracking control of a nonholonomic mobile robot[J]. IEEE Trans on Robotics and Automation, 2000, 16(5): 609-615.

[14] Das T, Kar I N: Design and implementation of an adaptive fuzzy logic-based controller for wheeled mobile robots[J]. IEEE Trans on Control Systems Technology, 2006, 14(3): 501-510

[15] Yue M, Hu P, Sun W: Path following of a class of non-holonomic mobile robot with underactuated vehicle body[J]. IET Control Theory & Applications, 2010, 4 (10): 1898-1904.

[16] Fierro R, Lewis F L: Control of a nonholonomic mobile robot: Backstepping kinematics to dynamics[J]. J of Robotic Systems, 1997, 14(3): 149-163.

[17] Tsai C C, Wu H L, Tai F C: Intelligent sliding-mode formation control for uncertain networked heterogeneous Mecanum-wheeled omnidirectional platforms[C]. Proc of the 20th IEEE International Conference on Systems, Man, and Cybernetics. Banffff: IEEE, 2017: 539-544.

[18] Tsai C C, Wu H L, Tai F C, et al: Distributed consensus formation control with collision and obstacle avoidance for uncertain networked omnidirectional multi-robot systems using fuzzy wavelet neural networks[J]. International Journal of Fuzzy Systems, 2016, 19(5): 1-17.

[19] Dong Ma, Liyuan Dong, Liling Wang, Hongrui Wang: Mobile robot RBF Neural Network daptive PD Tracking Control[J]. Control Engineering, 2020,27(12): 2092-2098.

[20] Alakshendra V, Chiddarwar S S: Adaptive robust control of mecanum-wheeled mobile robot with uncertainties[J]. Nonlinear Dynamics, 2017, 87(4): 2147-2169.

[21] Ma S G, Zhao J L, Ren C: Passivity-based active disturbance rejection control of an omni-directional mobile robot[J]. Control & Decision, 2018, 33(6): 1081-1086.

[22] Zhongting Sun, Jianjun Bai, Bingxu Chen, Yun Chen: Wheel mobile robot adaptive trajectory tracking control[J/OL]. Control Engineering, 1-6[2021-11-24]. j.cnki.kzgc.20200387.