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Research on Radial Hilbert Transform Based on Bessel-like Amplitude Modulation Spiral Phase Filter

Jun Chen, Chuantao Ma

School of Physics and Electronic Engineering, Taishan University, Taishan, Shandong 271021, China

Abstract

This paper proposes a method to improve the contrast of image edge enhancement by using Bessel-like amplitude modulation spiral phase filter, and proves the effect of this method in improving the contrast of image edge enhancement by calculation and simulation.

Keywords

Radial Hilbert Transform , Bessel-like Amplitude Modulation Spiral Phase Filter.

1. Introduction

Hilbert transform filtering is often used in image processing because it can selectively enhance the edges of the input image, but one-dimensional or two-dimensional Hilbert transform can only enhance the edges of the image along the horizontal or vertical direction. In 2000, Jeffrey A.。Davis et al. proposed a radial symmetric edge enhancement method[1]for two-dimensional images based on spiral phase filtering, that is, Radial Hilbert Transform. Experiments have proved that coherent spatial filtering using spiral phase devices can enhance the edge contrast of both amplitude-type objects[2-3]and phase-type objects[4-6].However, due to the sidelobe influence of the point spread function, the ordinary spiral phase filter will blur the image edge and reduce the contrast of edge enhancement.

In our previous work, we used the Bessel-like amplitude modulation spiral phase method to suppress optical vortex sidelobes, and gave the analytical solution[7]of the optimal modulation parameters. According to the previous viewpoint, since Bessel-like amplitude modulation can better suppress the sidelobe of optical vortex, that is, the sub-maximum in the point spread function, This method can also improve the contrast of the output image of radial Hilbert transform.

2. Point spread function of Bessel-like amplitude modulated spiral phase filter

For the spiral phase structure with low topological charge, the best effect can be achieved by using the Bessel-like function amplitude modulation of (n+1) order, but we have not given the diffraction light field expression of the spiral phase structure in this case. Here, we first deduce the Fraunhofer diffraction light field in this case.The transmittance function of the low-order Bessel-like amplitude modulation spiral phase filter can be expressed as:

$$\tau_n(r,\varphi)=circl\left(\frac{r}{R}\right)\frac{J_{n+1}(\alpha r)}{r}\exp(in\varphi) \qquad n=0,\pm1,\pm2,\cdots \tag{1}$$

In which(r,φ)is the polar coordinate on the filter plane, n is the topological charge of the spiral phase filter, R is the radius of the spiral phase filter, α is the amplitude modulation function, and

Jn+1(x) is the Bessel function of the first kind of n+1 order. the Fraunhofer diffraction field of Bessel-like modulation spiral phase filter can be obtained as follows:

$$\begin{aligned} E_n(\rho,\theta) &= (-i)^{n+1}\frac{k}{f}\exp(in\theta)\int_0^R J_{n+1}(\alpha r)J_n\left(\frac{k}{f}\rho r\right)dr \\ &= (-i)^{n+1}\frac{kR}{n!f}\left(\frac{k\alpha\rho R^2}{4f}\right)\exp(in\theta)\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{\alpha R}{2}\right)^{2m+1}}{m!(2m+2n+1)\Gamma(m+n+2)}\times \\ &\quad {}_1F_2\left[m+n+1, m+n+2, n+1, -\left(\frac{kR\rho}{2f}\right)^2\right] \end{aligned} \tag{2}$$

In which ${}_1F_2(a,b,c,x)$ is a hypergeometric function and $\Gamma(x)$ is a gamma function.

To realize the first-order radial Hilbert transform, we need to use Bessel-like amplitude modulation spiral phase filter with topological charge of 1, and its transmittance function is.

$$H_1(\rho,\theta) = circl\left(\frac{r}{R}\right)\frac{J_2(\alpha\rho)}{\rho}\exp(i\theta) \tag{3}$$

The optimal amplitude modulation parameter $\alpha=\gamma_{2,1}/R$, where $\gamma_{2,1}$ is the first root of the second-order Bessel function. According to formula (3), the point spread function of Bessel-like amplitude modulation spiral phase filter with topological charge of 1 can be obtained as follows:

$$\begin{aligned} h_1(r,\varphi) &= -\frac{\alpha k^2 R^3 r}{4f^2}\exp(i\varphi)\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{\alpha R}{2}\right)^{2m+1}}{m!(2m+3)\Gamma(m+3)}\times \\ &\quad {}_1F_2\left[m+2, m+3, 2, -\left(\frac{kR\rho}{2f}\right)^2\right] \end{aligned} \tag{4}$$

From equation (4), we can calculate the point spread function of Bessel-like amplitude modulation spiral phase filter.

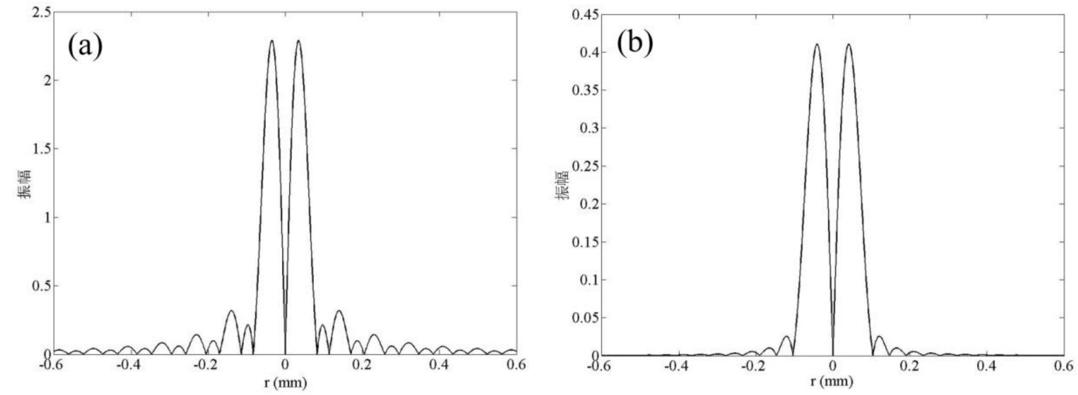

Fig. 1 (a)Radial distribution of point spread function, (b) Ordinary spiral phase plate (n=1,R=3mm),(b) Bessel amplitude modulated spiral phase plate (n=1,R=3mm,α=1.712mm^{-1}).

As shown in Figure 1, compared with the point spread function distribution curve of ordinary spiral phase filter, the point spread function of Bessel-like amplitude modulation spiral phase filter can suppress most sub-maxima, and in terms of the effect of suppressing high-frequency maxima, Bessel-like amplitude modulation spiral phase filter is superior to Laguerre-Gaussian filter.

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3. Simulation of radial Hilbert transform of Bessel amplitude modulated spiral phase filters

Let the input function be $g(r,\varphi)$, and use Bessel-like amplitude modulation spiral phase filter $H_1(\rho,\theta)$ to perform spectral filtering at its spectral position, and then pass through Fourier transform lens at the output end to obtain the result of image edge enhancement:

$$\tilde{g}(r,\varphi)=g(r,\varphi)*h_1(r,\varphi) \tag{5}$$

That is, the output function is the convolution of the input function and the point spread function of the Bessel-like amplitude modulation spiral phase filter.

Based on the above theoretical analysis, we simulated the radial Hilbert transform with the simplest amplitude circular aperture as an example, as shown in Figure 2. The simulation results of circular aperture (A) after ordinary spiral phase filtering and Bessel-like amplitude modulation phase filtering are respectively (B) and (C), which are given by corresponding (E) and (F) diagrams.

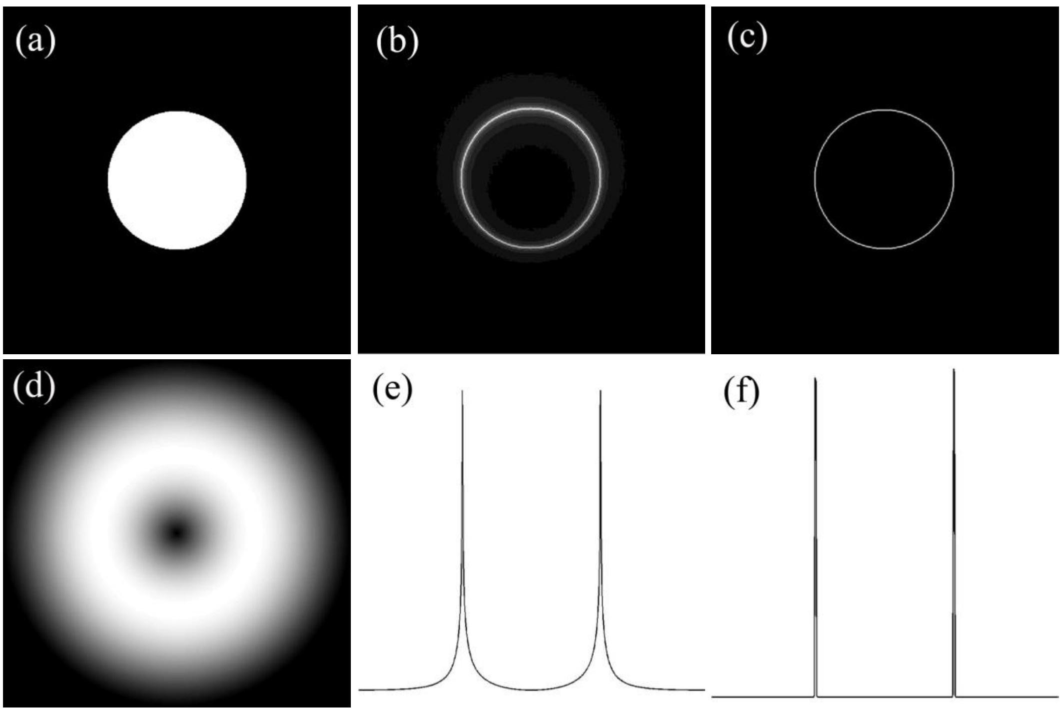
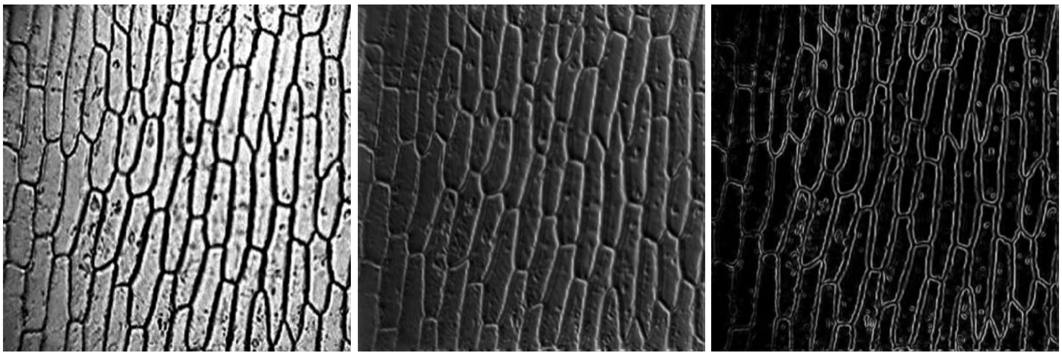


Fig. 2 Hilbert transform simulation results of circular aperture, (a) circular aperture, (b) filtering results of ordinary spiral phase filter, (c) filtering results of Bessel-like amplitude modulation spiral phase filter, (d) distribution of Bessel-like amplitude $J_2(\alpha r)/r$, (e) and (f) are those of (b) and (e) respectively.

From the one-dimensional light intensity distribution curve of the output image, it can be seen that there is noise at the edge of the image in the output result of ordinary spiral phase filter. When the two edges are relatively close, this noise will seriously affect the contrast of image edge enhancement, while the output image of Bessel-like amplitude modulation spiral phase filter obviously eliminates this noise.It is helpful to improve the contrast and resolution of radial Hilbert transform. In addition, we also use the image to simulate and achieve the same effect, as shown in Figure 3. Fig. 3 shows the radial Hilbert transform results of onion epidermal cell images collected under microscope. In Fig. 3(b),The Hilbert transform result of ordinary spiral phase filter has a certain relief effect due to noise interference, while the Bessel-like amplitude modulation spiral phase filter can completely and clearly outline the cell boundary structure, which greatly improves the contrast of image edge enhancement.



(a) (b) (c)

Fig. 3 edge enhancement results of onion epidermal cells, (a) images collected under bright field conditions of microscope, (b) ordinary spiral phase filtering results of fig. (a), and (c) Bessel-like amplitude modulation spiral phase filtering results of fig. (a).

From the above simulation results, it can be seen that the radial Hilbert transform based on Bessel-like amplitude modulation spiral phase filter can greatly improve the contrast of the image and eliminate the noise in the output image.

4. Conclusion

In this paper, we put forward the viewpoint of using Bessel-like amplitude modulation spiral phase filter to improve the contrast of edge enhancement of the output image of radial Hilbert transform, and proved the effect of this method in improving the contrast of edge enhancement image through calculation and simulation.

References

[1]. J. A. Davis, D. E. Mcnamara, D. M. Cottrell. Image processing with the radial Hilbert transform: theory and experiments. Opt. Lett. 2000, 25: 99-101.

[2]. Crabtree, J. A. Davis, I. Moreno. Optical processing with vortex-producing lenses. Appl. Opt. 2004, 43: 1360-1367.

[3]. Jesacher, S. Furhapter, S. Bernet, et al. Shadow Effects in Spiral Phase Contrast Microscopy. Phys. Rev. Lett. 2005, 94: 233902.

[4]. S. Furhapter, A. Jesacher, S. Bernet, et al. Spiral phase contrast imaging in microscopy. Opt. Express, 2005, 13: 689-694.

[5]. G. A. Swartzlander. Peering into darkness with a vortex spatial filter. Opt. Lett. 2001, 26: 497-499

[6]. G. Foo, D. M. Palacios, G. A. Swartzlander. Optical vortex coronagraph. Opt. Lett. 2005, 30: 3308-3310.

[7]. J. Chen, X.-C. Yuan, X. Zhao, Z. L. Fang, S. W. Zhu. Generalized approach to modifying optical vortices with suppressed sidelobes using Bessel-like functions. Opt. Lett. 2009, 34: 3289~3291.