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Design of synovial variable structure guidance law under the constraint of extremity intersection angle

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Abstract

The design of the guidance law is the key technology to achieve accurate impact or interception of the target. This paper designs a synovial variable structure guidance law under the constraint of the intersection angle. Firstly, the mathematical model under the constraint condition is introduced, and the state equation of the relative motion of the missile and the target is obtained. Secondly, according to the equation of state, the guidance law is designed in the longitudinal plane and the lateral plane respectively. By selecting the appropriate adaptive approach law, the guidance law that meets the requirements is designed. Finally, the result is simulated and verified to meet the constraints.

Keywords

Extremity intersection angle, Synovial variable structure, Guidance law, Adaptive approach law.

1. Introduction

The missile guidance system is a complex system with multi-variable, nonlinear, time-varying and various uncertainties such as model uncertainty, external interference and other factors. At the same time, with the advancement of technology, the working environment of missiles is becoming more and more complicated. , The target's mobility becomes stronger, so higher requirements are put forward for the design of the guidance law. Sliding mode variable structure control is a robust control method that can effectively deal with uncertain nonlinear systems [1]. It has great advantages in the design of the guidance law, and is aimed at the structural uncertainty and external interference of the system. The problem of uncertain factors can obtain satisfactory dynamic performance. In the process of designing the guidance law, under some specific conditions, not only the miss distance requirement must be met, but also the rendezvous angle constraint must be met. For example, it is hoped to hit a target with military value vertically, and it is hoped to use a large-angle dive to attack supersonic missiles. Therefore, it is of great significance to study this kind of guidance law with rendezvous angle constraints.

2. The mathematical description of the terminal interception problem

As we all know, zero line-of-sight angular velocity is the ideal state in missile interception, and the missed target can be guaranteed $\dot{q}=0$ to be zero. When colliding with the target, the overload and the angle of attack are both small, and the attitude angle is the sum of the ballistic inclination and the angle of attack, it can be considered that the missile attitude angle of myopia is equal to the ballistic inclination [2]. Since the target's ballistic inclination and deflection angle are constantly changing, the angle of sight of the projectile when hitting the target can be controlled to indirectly satisfy the rendezvous angle constraint.

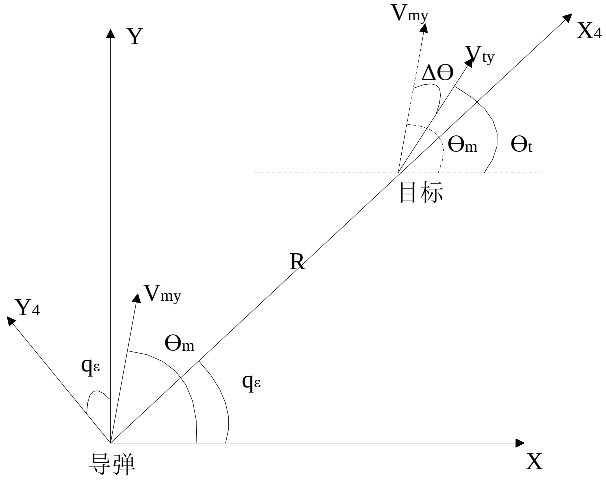

Figure 1: The relative movement of the projectile in the longitudinal plane

Let $q_{\varepsilon f}$ be the expected inclination of the line of sight when the missile is about to hit the target, then

$$\mathrm{v}_{ty}\sin\left(\theta_t - q_{\varepsilon f}\right) - \mathrm{v}_{my}\sin\left(\theta_m - q_{\varepsilon f}\right) = 0 \tag{1}$$

In the formula, v_{ty}, v_{my} and respectively represent the speed of the target and the missile in the longitudinal plane, which are derived as

$$q_{\varepsilon f} = \theta_t\left(t_f\right) + \tan^{-1}\left(\frac{\sin\Delta\theta*}{\cos\Delta\theta* - \rho}\right) \tag{2}$$

Where $\Delta\theta^*$ is the desired relative inclination angle of the end; $\rho = \frac{\mathrm{v}_{ty}}{\mathrm{v}_{my}}$ is the velocity ratio in the longitudinal plane. Since the velocity of the missile and the target changes little in the terminal guidance section, assuming that the velocity of the missile and the target is unchanged, ρ can also be approximated as a constant value.

In the same way, in the lateral plane, let $q_{\beta f}$ be the expected deflection angle of the line of sight when the missile is about to hit the target, we can get

$$q_{\beta f} = \psi_{vt}\left(t_f\right) + \tan^{-1}\left(\frac{\sin\Delta\psi*}{\cos\Delta\psi* - \rho'}\right) \tag{3}$$

The analysis shows that, as long as the expected end intersection angle and the target ballistic inclination (ballistic deflection angle) are given, it can be converted into the corresponding line-of-sight inclination angle (line-of-sight declination) constraint.

In summary, the relative motion of the missile and the target can be derived from the following state equations:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ a_2 \end{bmatrix} u_y + \begin{bmatrix} -1/V_{ty} \\ a_3 \end{bmatrix} a_{ty} \\ \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & b_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u_z + \begin{bmatrix} -1/V_{tz} \\ b_3 \end{bmatrix} a_{tz} \end{cases} \tag{4}$$

3. Synovial variable structure guidance law design

3.1. Design of the longitudinal in-plane guidance law

In order to make the system state equation (4) insensitive to parameter perturbation and disturbance changes, we consider using variable structure control theory to design the guidance law. Based on the principle of quasi-parallel approach [3], it is hoped that $\dot{q}_\varepsilon$ will tend to zero during the guidance process. At the same time, considering the end intersection angle constraint, the sliding mode is selected as

$$s_1(\mathrm{t})=\frac{k_1\left|\dot{R}(t)\right|}{R(t)}x_1+k_2x_2 \tag{5}$$

We can apply the law of reaching when deriving the controller. The system state equation (4) is a linear time-varying system. It is not feasible to apply the general exponential reaching law, constant velocity reaching law, etc. [4]. In order to make the system state not only reach the sliding mode surface, but also In the process of sliding mode surface to ensure that the system has good dynamics, we consider the adaptive sliding mode reaching law [5].

The general expression of the adaptive reaching law is

$$\begin{cases}\dot{s}(t)=-f\left[s(t),p(t)\right]-\varepsilon\left[p(t)\right]\mathrm{sgn}\left[s(t)\right],\varepsilon>0\\ f\left[0,p(t)\right]=0,s(t)f\left[s(t),p(t)\right]>0,if\ s(t)\neq 0\end{cases} \tag{6}$$

In the formula, $p(t)$ represents the parameters of the system.

The adaptive reaching law designed for the system state equation (4) above is

$$\dot{s}_1(\mathrm{t})=-k_3\frac{\left|\dot{R}(t)\right|}{R(t)}s_1(t)-\frac{\varepsilon_1\,\mathrm{sgn}\left[s_1(t)\right]}{R(t)} \tag{7}$$

In the formula, k_3, ε_1 both are constants greater than zero.

The reaching law of equation (7) has strong adaptability: because the change $\left|\dot{R}(t)\right|$ is not large in the process of terminal guidance, when the projectile and target are relatively far away, that is, when $R(t)$ is relatively large, the approaching sliding mode is appropriately slowed down. Velocity; when $R(t)$ approaches zero, the approach velocity is increased rapidly to ensure that $\dot{q}_\varepsilon(t)$ does not diverge, so that the missile has a higher hit accuracy. The application of adaptive theory to adjust the approach velocity can effectively weaken the sliding mode jitter [6].

Differentiate equation (5), we get

$$\dot{s}_1(\mathrm{t})=k_1\frac{\left[\left|\ddot{R}(t)\right|x_1+\left|\dot{R}(t)\right|\dot{x}_1\right]R(t)-\left|\dot{R}(t)\right|R(t)x_1}{R^2(t)}+k_2\dot{x}_2 \tag{8}$$

Substituting formula (8) into (7), we get

$$-k_3\frac{\left|\dot{R}(t)\right|}{R(t)}s_1(t)-\frac{\varepsilon_1\,\mathrm{sgn}\left[s_1(t)\right]}{R(t)}=k_1\frac{\left[\left|\ddot{R}(t)\right|x_1+\left|\dot{R}(t)\right|\dot{x}_1\right]R(t)-\left|\dot{R}(t)\right|R(t)x_1}{R^2(t)}+k_2\dot{x}_2 \tag{9}$$

In addition, the sliding mode variable structure is robust to the perturbation of system parameters [7], and considering that $\dot{R}(t)<0$, and $\dot{R}(t)$ is regarded as a constant,then $\ddot{R}(t)=0$, the above formula can be simplified as

$$k_3\frac{\dot{R}(t)}{R(t)}s_1(t)-\frac{\varepsilon_1\,\mathrm{sgn}\left[s_1(t)\right]}{R(t)}=k_1\frac{\dot{R}^2(t)x_1-\dot{R}(t)R(t)\dot{x}_1}{R^2(t)}+k_2\dot{x}_2 \tag{10}$$

Substituting equations (4) and (5) into (10) to get

$$a_{my}=\frac{(k_1k_3+k_1)\dot{R}^2(t)}{k_2R(t)}x_1-\frac{(k_1+2k_2+k_2k_3)\dot{R}(t)}{k_2}x_2+\frac{k_1\dot{R}(t)\dot{\theta}(t)}{k_2}+a_{ty}+\frac{\varepsilon_1\,\mathrm{sgn}\left[s_1(t)\right]}{k_2} \tag{11}$$

According to the second method of Lyapunov, take the Lyapunov function as $V = s_1^2(t)/2$. Derivation of this function with respect to time [8], and considering the system state equation, we get

$$\begin{aligned}\dot{V}(t) &= s_1(t)\dot{s}_1(t) = -k_3\frac{|\dot{R}(t)|}{R(t)}s_1^2(t) - \frac{\varepsilon_1 \operatorname{sgn}[s_1(t)]s_1(t)}{R(t)} \\ &= -k_3\frac{|\dot{R}(t)|}{R(t)}s_1^2(t) - \frac{\varepsilon_1|s_1(t)|}{R(t)}\end{aligned} \tag{12}$$

It can be seen from the above formula that when $s_1(t) \neq 0$ is $\dot{V}(t) < 0$, the Lyapunov stability criterion was met, which shows that the state variable $x_1(t)$, $x_2(t)$ can tend to zero in a finite time, and meet the requirements of the expected end relative tilt angle and zero miss distance.

3.2. Design of lateral in-plane guidance law

According to the symmetrical relationship of the two planes, the guidance law in the lateral plane can be obtained in the same way.

Take the sliding mode as

$$s_2(\mathrm{t}) = \frac{k_4|\dot{R}_1(t)|}{R_1(t)}y_1 + k_5 y_2 \tag{13}$$

Among, $k_4 = const > 0, k_5 = const > 0$.

Select the adaptive reaching law as

$$\dot{s}_2(\mathrm{t}) = -k_6\frac{|\dot{R}_1(t)|}{R_1(t)}s_2(t) - \frac{\varepsilon_2 \operatorname{sgn}[s_2(t)]}{R_1(t)} \tag{14}$$

Among, $k_6 = const > 0, \varepsilon_2 = const > 0$.

In the same way, the lateral in-plane guidance law can be obtained as

$$a_{mz} = -\frac{(k_4k_6 + k_4)\dot{R}_1^2(t)}{k_5R_1(t)}y_1 + \frac{(k_4 + 2k_5 + k_5k_6)\dot{R}_1(t)}{k_5}y_2 - \frac{k_4\dot{R}_1(t)\dot{\psi}_{vt}(t)}{k_5} - a_{tz} - \frac{\varepsilon_2 \operatorname{sgn}[s_2(t)]}{k_5} \tag{15}$$

Equations (11) and (15) contain switch function terms, which need to switch the control variables. However, in the actual system, the switching of the control quantity is not completed instantaneously. There is always a certain time delay, which will inevitably cause jitter. If the amplitude of jitter is too large, it may cause certain harm. In order to avoid this phenomenon, We need to smooth the non-continuous switching function [9]. We can use high-gain continuous functions $s_1(t)/[|s_1(t)| + \delta]$ and $s_2(t)/[|s_2(t)| + \delta_1]$ to replace the sum respectively, where $\operatorname{sgn} s_1(t)$ and $\operatorname{sgn} s_2(t)$ are both very small positive numbers. The smoothed SASMG is

$$\begin{cases} a_{my} = \dfrac{(k_1k_3 + k_1)\dot{R}^2(t)}{k_2R(t)}x_1 - \dfrac{(k_1 + 2k_2 + k_2k_3)\dot{R}(t)}{k_2}x_2 + \dfrac{k_1\dot{R}(t)\dot{\theta}(t)}{k_2} + a_{ty} + \dfrac{\varepsilon_1 s_1(t)}{k_2[|s_1(t)| + \delta]} \\ a_{mz} = -\dfrac{(k_4k_6 + k_4)\dot{R}_1^2(t)}{k_5R_1(t)}y_1 + \dfrac{(k_4 + 2k_5 + k_5k_6)\dot{R}_1(t)}{k_5}y_2 - \dfrac{k_4\dot{R}_1(t)\dot{\psi}_{vt}(t)}{k_5} - a_{tz} - \dfrac{\varepsilon_2 s_2(t)}{k_5[|s_2(t)| + \delta_1]} \end{cases} \tag{16}$$

4. Simulation verification

The simulation model of the missile guidance system is shown in Figure 2. In the simulation of the missile guidance system, the initial state of the missile and the target is first given, including the initial position and initial speed. The missile-target relative movement module is based on the movement information of the missile and the target. The required guidance information is calculated, and the guidance module is sent to the autopilot to control the missile to change its trajectory after obtaining the guidance instruction, and finally accurately intercept and hit the target.

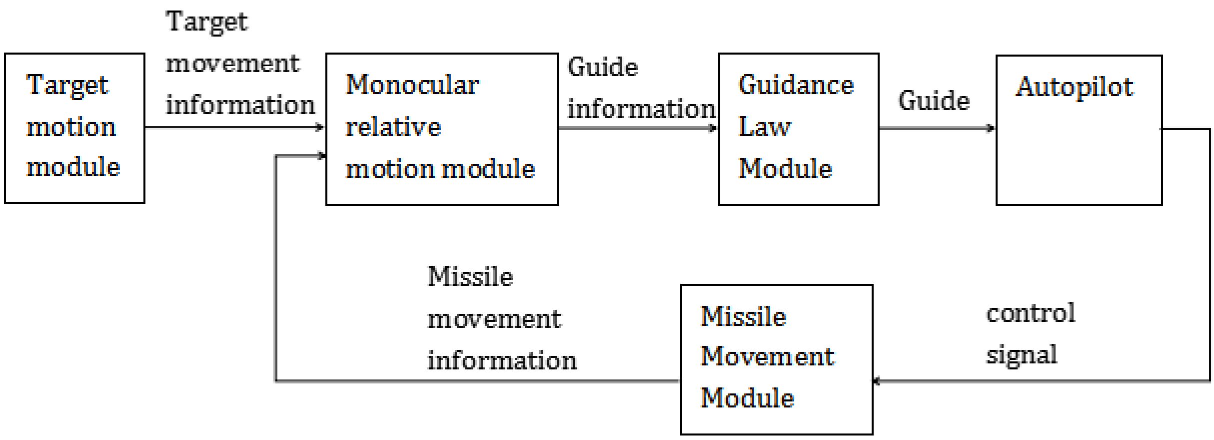

Figure 2: Simulation model diagram of missile guidance system

Assuming the initial conditions of terminal guidance: set the parameters $k_1=2, k_2=1, k_3=1$; $k_4=2, k_5=1, k_6=1$。 missile speed $V_m=800m/s$, target speed $V_t=300m/s$, initial relative distance of missile and eye $R(0)=4000m$, initial line of sight inclination angle of missile and eye $q_\varepsilon=15^o$, initial line of sight deflection angle of missile and eye $q_\beta=30^o$, The anti-vibration factor $\delta=0.02$, $\varepsilon=200$, the expected deflection angle and the inclination angle of the line of sight are all set to zero [10]. The simulation result is Case 1, as shown in Figure 3-5. Figure 4 is the relative movement curve of the missile and eye in the three-dimensional space. It can be seen from the figure that the missile can not only hit the target, but the guidance end meets the end intersection angle of zero; Figure 5 is the curve of the relative distance between the missile and eye in the longitudinal plane. It can be seen from the figure that the relative distance between the projectile and the target becomes zero at approximately 9.24s; Figure 6 is the curve of the relative inclination and the relative declination of the projectile and the target. From the figure, it can be seen that the relative inclination and relative declination of the projectile and the target become zero at 9.24s. In summary, the designed guidance law meets the performance index requirements.

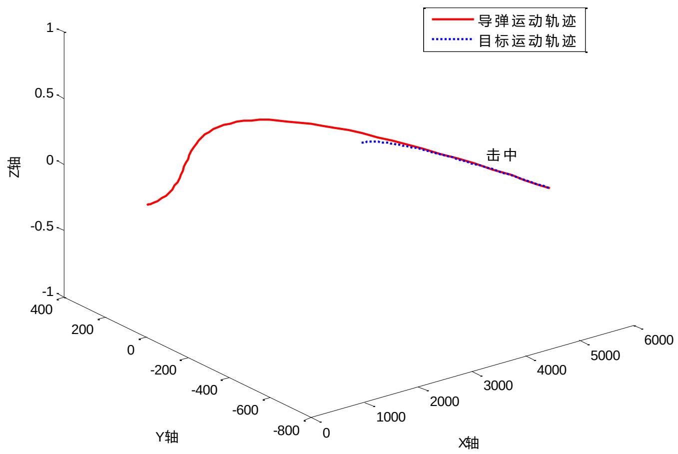

Figure 3: Relative motion curve of missile-target

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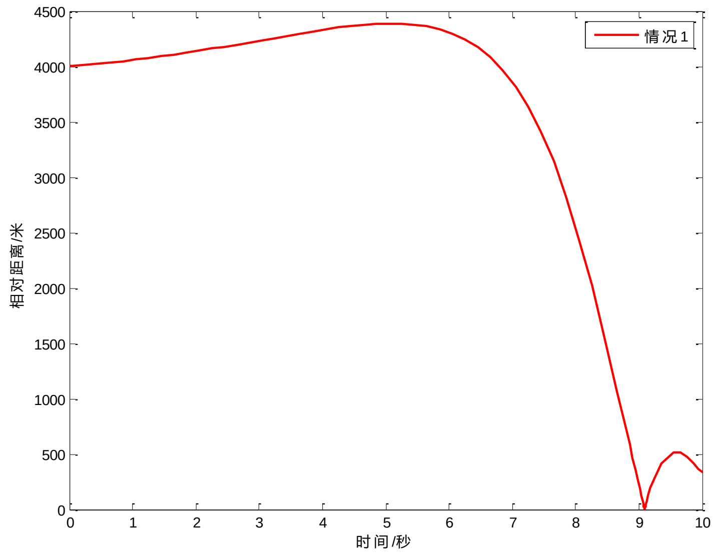

Figure 4: Curve of relative distance change of projectile and target

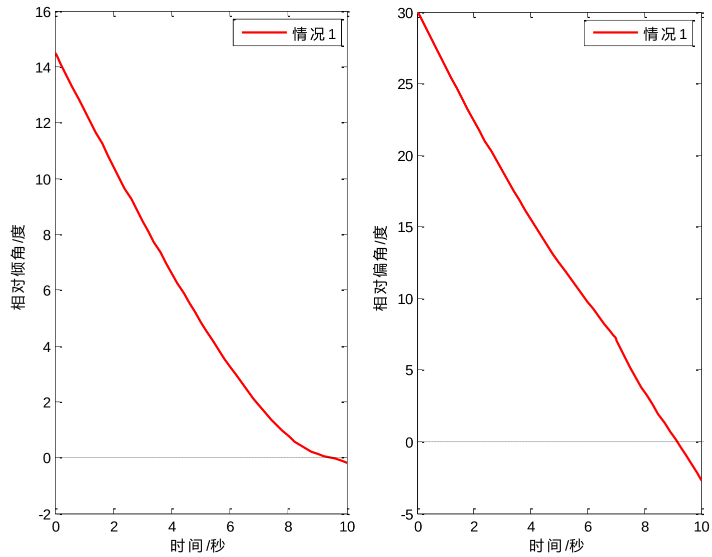

Figure 5: Curves of relative inclination and relative deflection of projectile and eye

5. Conclusion

This paper designs a guidance law that hopes to hit the target with the desired intersection angle. First, the state equation of the relative motion of the projectile and the target under the restriction of the end intersection angle is derived. The state equation is designed to be adaptive in the longitudinal plane and the lateral plane. At the same time, according to the second method of Lyapunov, it is proved that the stability of the system after adding this law can meet the requirements of the expected end intersection angle and zero miss distance. Finally, the simulation verification shows that the required performance can be met. index. This article only proves the feasibility theoretically, but its application in practical guidance needs further study.

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