Forecast of Container Demand Based on Grey Markov Model
Haojie Du, Wei Liu
School of Shanghai Maritime University, Shanghai 201306, China

Abstract
As a kind of “door to door” transport, container transport has become the main mode of transport between various modes of transport and even the international transport of goods. With the increasing of container demand, it is of great theoretical and practical significance to predict container demand. Since the container demand data collected in this paper is small, the grey prediction model is firstly determined when selecting the prediction method. Moreover, container demand data are affected by many factors. This paper did not choose a single method when selecting the method, but established a grey Markov model to predict container demand based on the advantages of Markov prediction model. The results show that the prediction results and accuracy are good, and the model is suitable for the prediction of container demand.

Keywords
Container demand; Grey Markov prediction model; Prediction accuracy.

1. Introduction
As we all know, railways play a mainstay role in the development of China's national economy. China’s railway lines will soon be built into an eight horizontal and eight vertical road network. By then, China’s transportation routes from the capital to the frontiers and from the inland to the coast will be more abundant and developed. The railway can transport all kinds of materials and materials needed for our production and life into the hands of people, and meet the needs of the people in terms of production and life. Railway transportation can safely transport large steel and coal to large enterprises for production and manufacturing, and can also transport clothing and grain to cities for people’s daily use.

Container transportation also plays a pivotal role in railway transportation. Container transportation is a transportation method that can realize "door-to-door" transportation. Container transportation has many characteristics and advantages. It can facilitate cargo owners to carry, load, unload, and move goods, and is convenient for delivery. It can also make full use of the load of railway vehicles. It can accelerate the turnover of railway transportation vehicles and ensure the transportation quality of goods. On this basis, it can also improve the transportation quality of goods and improve transportation efficiency, which can realize the modernization of railway transportation. Container transportation is already the main means of transportation for the conversion of various modes of transportation to the extent that various countries handle containerized cargo intermodal transportation. The development of container transportation has brought about the standardization and serialization of containers, and has also promoted the research and development of various new special vehicles such as railway container vehicles and container road-rail dual-use vehicles.

The forecast of container demand affects the organization of railway container transportation and is related to the railway container transportation plan. The professional terms such as the number of containers used, the number of non-used containers, the number of empty containers and the number of empty containers in the transportation plan are closely related to the demand for containers. A reasonable forecast of the demand for containers can determine the number of containers and optimize the container reasonably. Play a role in transport...
organization. The demand for containers is also related to the design of the container yard. When arranging containers in the container yard, full consideration should be given to the amount of containers used, which will affect the area of the container yard. The reasonable arrangement of various containers can speed up loading and unloading operations and improve transportation efficiency.

From an economic point of view, supply is determined by demand. The quantity of containers supplied and the quality characteristics of the supplied containers are determined to a certain extent by the quantity of container demand, in order to facilitate customers to carry out container transportation and satisfy customers' demand for containers. The installation of the railway freight center container station came into being. The number of containers required by customers is a major basis for the development and construction of the container center station. The transportation system of the container central station is not an ordinary and simple transportation system. It is a large system that combines the geographical factors of the railway line location, the transportation line, the location of the container yard and the installation of related loading and unloading equipment. For the transportation routes, container yards and related loading and unloading equipment involved in the above-mentioned reasonable layout, the first thing to bear the brunt is to accurately predict the number of containers required. Therefore, the key role in the reasonable planning of the container yard design is to accurately control the number of containers required.

Control the precise forecast of container demand and the development trend of container demand. These tasks are of great significance to the predictability of railway transportation organization and the optimization of transportation organization. Precisely predict the number of container requirements. When organizing and managing the container transportation process, managers can understand the changes in future container market demand in a timely manner and reasonably control the container transportation organization, thereby being able to perform operations on the entire container station. Make reasonable decisions. The demand for containers is closely related to the cost of container transportation and the profitability of container transportation. The accuracy of the overall benefit evaluation will also be affected by the accuracy of container demand forecasting.

2. Literature Review

The activities of people to infer and determine the development of certain things or phenomena in the future based on existing knowledge methods that have been familiar to people, we call it prediction. Specifically, people use a variety of qualitative and quantitative methods to make scientific predictions of the possible state or level of things that can be reached in the future according to the changing laws of the past state of things, according to the current state of movement and change.

Some scholars have begun to develop from a single prediction method to a combination of various methods, and the statistical system and scientific concepts of prediction are also at the forefront of the subject. To put it simply, the following examples can be cited: Chiou HK, Tzeng GH, etc. (2002)[1] Based on the research of the GM(1,1) forecasting model, the inventory of logistics is predicted, and accurate forecast results are obtained. Tang W, Li W (2005)[2] established a linear regression prediction model that can support vector machines, and also introduced a time series to combine with it, and finally improved the model through the analysis of examples. Hu Y (2017)[3] proposed a model to further improve the prediction accuracy of the GM (1,1) prediction model: the gray prediction model based on genetic algorithm, Adamowski (2008)[4] and Wang G (2013)[5] construct prediction models through neural networks, etc. These algorithms have good linear and nonlinear calculation processing capabilities.
With the development of prediction methods and the popularization of related theoretical knowledge, Chinese researchers have also paid attention to and researched related theoretical methods and models of prediction. Wang Huiwen, Meng Jie (2007) [6] established a multiple linear regression model based on historical sample data; Liu Hong, Wang Ping (2007) [7] summarized linear forecasting methods based on time series; Lu Yang (2017) [8] established a gray scale with few data Linear programming prediction model; Shi Qingdong(2017) [9] used the Grey Markov Model to predict occupational diseases in ten years. Song Xiaozhen (2019) [10] and others used this model to predict coal production. The publication of these research results has enabled us to have more abundant and easily available reference materials for demand forecasting.

The research direction of Chinese researchers on demand forecasting focuses on the accuracy of algorithm calculations, generally based on the data of relevant national official websites and some national macro policies to predict, and artificially determine the relevant parameters of demand forecasting. With the rapid development of China’s economy and the prosperity of the transportation industry, we have begun to use a combination of qualitative and quantitative analysis to predict relevant demand. In the West, compared to the domestic ones, Western researchers use the demand theory in Western economics to start with factors that affect demand, and visually show the changes in demand through changes in the demand curve. There are differences in forecasting methods and thinking between China and foreign countries. What we need is a forecasting method that is more suitable for our own thinking. We plan to focus on domestic accurate algorithms. Research at home and abroad has shown that under certain circumstances, the combination of demand forecasting models The prediction accuracy is generally higher than that of a single prediction model. For example, the gray neural network is very suitable for the fitting process of small and non-linear data. If the gray neural network model is used to predict container demand, the prediction accuracy will be considerable.

3. Demand forecasting model construction

3.1. GM(1,1) prediction model

GM(1,1) predictive model means that the variable in the equation is a single-sequence gray first-order differential equation, which describes the regular changes of discrete data. By defining the gray derivative and gray differential equation, the approximate differential of the discrete data is established. equation. GM (1, 1) model when the original data has a certain exponential change law, the prediction accuracy is considerable.

3.1.1. Basic form

Set the variable \( X^{(0)} = \{X^{(0)}(i), i = 1, 2, ..., n\} \) as the original data sequence. To establish a gray prediction model, first perform \( X^{(0)} \) Accumulation (1-AGO), generate an accumulation sequence: \( X^{(0)} = \{X^{(0)}(i), i = 1, 2, ..., n\} \), and:

\[
X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i) = X^{(1)}(k - 1) + X^{(0)}(k)
\]

(1)

For \( X^{(1)} \), the whitening differential equation can be established:

\[
\frac{dX^{(1)}}{dt} + aX^{(1)} = u
\]

(2)

The solution of the whitening equation is:

\[
\hat{X}^{(1)}(k + 1) = \left( X^{(0)}(1) - \frac{u}{a} \right) e^{-ak} + \frac{u}{a}
\]

(3)

Or:
Where $k$ is the time series, and the unit is year, quarter, or month.

### 3.1.2. Class ratio judgement

The sequence $X^{(0)} = \{X^{(0)}(i), i = 1, 2, \ldots, n\}$ has the class ratio of:

$$\sigma(k) = \frac{X^{(0)}(k)}{X^{(0)}(k-1)}, \quad k = 2, 3, \ldots, n$$  \hfill (5)

The smooth ratio of the sequence is:

$$\rho(k) = \frac{X^{(k)}}{\sum_{i=1}^{k-1} X^{(0)}(i)}, \quad k = 2, 3, \ldots, n$$ \hfill (6)

If class ratio of the sequence $\sigma(k)$ satisfies:

1. For any number $k$, $\sigma(k) \in (0,1)$, we say $X(0)$ that has a negative gray index law.
2. For any number $k$, $\sigma(k) \in (1, b)$, where $b$ is a real number greater than 1, it is said that the sequence $X(0)$ has a positive gray index law.
3. For any number $k$, $\sigma(k) \in (a, b)$, $b-a=\Delta$, then the sequence $X(0)$ is said to have a gray index law with an absolute gray level of $\Delta$.
4. When $\Delta < 0.5$, it is said that $X(0)$ has a quasi-exponential law.

If the smooth ratio $\rho(k)$ of sequence $X(0)$ satisfies:

1. $\frac{\rho(k+1)}{\rho(k)} < 1, \quad k = 2, 3, \ldots, n$
2. $\rho(k) \in [0, \delta], \quad k > 3, 4, \ldots, n$
3. $\delta < 0.5$

Call $X(0)$ the quasi-smooth sequence.

### 3.1.3. Calculation of parameters

If the parameter sequence is $\hat{e} = \left[\hat{a}, \hat{u}\right]^T$, can be obtained by the following formula:

$$\hat{e} = (B^T B)^{-1} B^T Y_n$$ \hfill (7)

In the formula:

- $B$-data array;
- $Y_n$-the data column.

The structure of the matrix is as follows:

$$B = \begin{bmatrix}
-\frac{1}{2} (X^{(0)}(1) + X^{(0)}(2)) & 1 \\
-\frac{1}{2} (X^{(0)}(2) + X^{(0)}(3)) & 1 \\
\vdots \\
-\frac{1}{2} (X^{(0)}(n-1) + X^{(0)}(n)) & 1
\end{bmatrix}$$ \hfill (8)

$$Y_n = (X^{(0)}(2), X^{(0)}(3), \ldots, X^{(0)}(n))^T$$ \hfill (9)

\[\hat{X}^{(1)}(k) = (X^{(0)}(1) - \frac{u}{a}) e^{-\sigma(k-1)} + \frac{u}{a}\] \hfill (4)
3.1.4. Calculation of parameters
Since the GM model obtains a cumulative amount, the predicted value at \( k \in \{n+1, n+2, \ldots \} \), The data \( \hat{X}^{(1)}(k+1) \) obtained from the GM model must be inversely generated, that is \( \hat{X}^{(0)}(k + 1) \), the cumulative and subtractive generation is restored to before it can be used.

\[
\hat{X}^{(1)}(k) = \sum_{i=1}^{k} \hat{X}^{(0)}(i) = \sum_{i=1}^{k-1} \hat{X}^{(0)}(i) + \hat{X}^{(0)}(k) \tag{10}
\]

\[
\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \sum_{i=1}^{k-1} \hat{X}^{(0)}(i) \tag{11}
\]

Because \( \hat{X}^{(1)}(k - 1) = \sum_{i=1}^{k-1} \hat{X}^{(0)}(i) \), so:

\[
\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k - 1) \tag{12}
\]

3.1.5. Model checking
① Residual error test
The residual test compares the original data with the predicted data, and observes the magnitude of the error value. Suppose the original sequence:

Predicted value is: \( \hat{X}^{(0)} = \{\hat{X}^{(0)}(i) \mid i = 1, 2, \ldots, n\} \)

The relative error is:

\[
\xi(k) = \frac{z(k)}{\hat{X}^{(0)}(k)} \times 100\% \tag{13}
\]

The residual sequence is:

\[
z(k) = \hat{X}^{(0)}(k) - \hat{X}^{(0)}(k) \tag{14}
\]

Where \( \xi(k) \) is not more than 10%.

The average relative error is:

\[
\overline{\varepsilon} = \frac{1}{n} \sum_{k=1}^{n} |\xi(k)| \tag{15}
\]

Model accuracy:

\[
P = (1 - \overline{\varepsilon}) \times 100\% \tag{16}
\]

② Posterior error test:
The posterior error test is based on the probability distribution of the residuals. Let the variance of the original sequence \( \{X(0)(k)\} \) be:

\[
S^2 = \frac{1}{n} \sum_{k=1}^{n} (X(0)(k) - \overline{X})^2 \tag{17}
\]

And,

\[
\overline{X} = \frac{1}{n} \sum_{k=1}^{n} X(0)(k) \tag{18}
\]

The variance of the residual sequence \( z(k) \) is:
\[ S_2^2 = \frac{1}{n} \sum_{k=1}^{n} (z(k) - \bar{z})^2 \]  \hspace{1cm} (19)

And,

\[ \bar{z} = \frac{1}{n} \sum_{k=1}^{n} |z(k)| \]  \hspace{1cm} (20)

The posterior difference ratio:

\[ C = \frac{S_2^2}{S_1^2} \]  \hspace{1cm} (21)

According to the calculated values of P and C, the accuracy can be divided into four levels, see Table 1.

<table>
<thead>
<tr>
<th>Accuracy class</th>
<th>P</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>&gt;0.95</td>
<td>≤0.35</td>
</tr>
<tr>
<td>Level 2</td>
<td>&gt;0.8</td>
<td>≤0.5</td>
</tr>
<tr>
<td>Level 3</td>
<td>&lt;0.7</td>
<td>≤0.65</td>
</tr>
<tr>
<td>Level 4</td>
<td>≤0.7</td>
<td>&gt;0.65</td>
</tr>
</tbody>
</table>

### 3.2. Markov prediction model

The Markov chain prediction model reflects the transfer process of complex systems through the Markov process.\[11\]

#### 3.2.1. Divide the state interval

First, calculate the relative value between the actual value and the predicted value of GM(1,1): \( Q = \frac{x(t)}{\tilde{x}(t)} \). Then divide the interval reasonably according to the relative value: \( E_i = [Q_{i1}, Q_{i2}], i = 1, 2, ..., k \), \( Q_{i1} \) and \( Q_{i2} \) are the upper and lower limits of the relative value respectively. The interval can be divided equally according to the number of relative values and the maximum and minimum values: \( N = \frac{Q_{max} - Q_{min}}{n} \), \( N \) is the divided interval distance, and \( n \) is the number of relative values.

#### 3.2.2. Calculate the state transition matrix

According to the number of times \( n_{ij} \) \( (k) \), the system transitions from state \( E_i \) to state \( E_j \) through \( k \) steps, calculate the state transition probability matrix: \( P_{ij} = \frac{n_{ij}(k)}{n_i} \).

\[
P_k = \begin{bmatrix}
p_{11k} & p_{12k} & \cdots & p_{1nk} \\
p_{21k} & p_{22k} & \cdots & p_{2nk} \\
\vdots & \vdots & \ddots & \vdots \\
p_{n1k} & p_{n2k} & \cdots & p_{nnk}
\end{bmatrix}
\]
3.2.3. Test Markov

Let \( \chi^2 \) be the test statistic defined as:

\[
\chi^2 = \sum_{i,j=1}^{n} f_{ij} \left( \frac{P_{ij}}{P_i} \right)^2.
\]

If the statistics obey \((n - 1)^2\), under the selected confidence level \(\alpha\), \(\chi^2 > \chi^2_{\alpha}(n - 1)^2\), then the statistic has Markov property, and Markov prediction can be performed on the sequence of numbers.

Through the test of Markov property, combined with the Markov state transition probability matrix, calculate the state interval \([Q_{i1}, Q_{i2}]\) where the predicted value of GM(1,1) is located, then the correction of the gray Markov predicted value is:

\[
\hat{x}(t) = \frac{1}{2} (Q_{i1} + Q_{i2}) \cdot X(t).
\]

4. GM(1,1) prediction

In the process of collecting data, we must fully consider the availability of data, and the data used reflects the theoretical knowledge on which it is based to a certain extent. When calculating the neural network model, in addition to the data of container demand, we also need data related to the following indicators: regional GDP (X1), primary industry added value (X2), secondary industry Value-added (X3), total sales of consumer goods (X4), Lanzhou GDP (X5), fixed asset investment in transportation, storage and postal industry (X6), and railway freight turnover (X7). These data come from the national data in the China Statistics Network. We reflect these data in Table 2:

<table>
<thead>
<tr>
<th>Year</th>
<th>X1 (Billion)</th>
<th>X2 (Billion)</th>
<th>X3 (Billion)</th>
<th>X4 (Billion)</th>
<th>X5 (Billion)</th>
<th>X6 (Billion ton kilometers)</th>
<th>Container demand(TEU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>3166.82</td>
<td>462.27</td>
<td>1470.34</td>
<td>1023.6</td>
<td>846.28</td>
<td>105.82</td>
<td>1120.06</td>
</tr>
<tr>
<td>2009</td>
<td>3387.56</td>
<td>497.05</td>
<td>1527.24</td>
<td>1183.0</td>
<td>926.00</td>
<td>145.48</td>
<td>1129.76</td>
</tr>
<tr>
<td>2010</td>
<td>4120.75</td>
<td>599.28</td>
<td>1984.97</td>
<td>1435.5</td>
<td>1100.40</td>
<td>192.00</td>
<td>1239.76</td>
</tr>
<tr>
<td>2011</td>
<td>5020.37</td>
<td>678.75</td>
<td>2377.83</td>
<td>1772.9</td>
<td>1360.30</td>
<td>247.13</td>
<td>1389.76</td>
</tr>
<tr>
<td>2012</td>
<td>5650.20</td>
<td>780.50</td>
<td>2600.09</td>
<td>2064.4</td>
<td>1563.80</td>
<td>301.85</td>
<td>1457.06</td>
</tr>
<tr>
<td>2013</td>
<td>6330.69</td>
<td>844.69</td>
<td>2745.35</td>
<td>2368.8</td>
<td>1776.28</td>
<td>428.50</td>
<td>1550.75</td>
</tr>
<tr>
<td>2014</td>
<td>6836.82</td>
<td>900.76</td>
<td>2926.45</td>
<td>2668.3</td>
<td>2000.94</td>
<td>793.74</td>
<td>1522.86</td>
</tr>
<tr>
<td>2015</td>
<td>6790.32</td>
<td>954.09</td>
<td>2494.77</td>
<td>2907.2</td>
<td>2095.99</td>
<td>814.89</td>
<td>1313.62</td>
</tr>
<tr>
<td>2016</td>
<td>7200.37</td>
<td>983.39</td>
<td>2515.56</td>
<td>3184.4</td>
<td>2264.23</td>
<td>1100.04</td>
<td>1220.35</td>
</tr>
<tr>
<td>2017</td>
<td>7459.90</td>
<td>859.75</td>
<td>2561.79</td>
<td>3426.6</td>
<td>2523.54</td>
<td>956.64</td>
<td>1390.72</td>
</tr>
</tbody>
</table>
4.1. Class ratio and smooth ratio

Before grey forecasting, the time series of container demand from 2008 to 2017 should be judged by grade ratio to judge whether it meets the requirements of establishing grey forecasting model.

Combining the formula of class ratio discrimination, the calculated grade ratios under different k values are shown in Table 3:

<table>
<thead>
<tr>
<th>k</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class ratio</td>
<td>1.00</td>
<td>0.97</td>
<td>1.03</td>
<td>1.09</td>
<td>1.06</td>
<td>1.08</td>
<td>1.04</td>
<td>1.13</td>
<td>1.01</td>
</tr>
</tbody>
</table>

According to the second item in the condition of grade ratio judgment, it is concluded that for any k=2…10, there is $\sigma(k) \in (1,1.13)$ where b=1.13>1, it can be seen that the current sequence has a positive Grey index law.

According to the third item in the judgment condition of grade ratio, it can be concluded that for any k=2…10, there is $\sigma(k) \in (0.97, 1.13)$, where a=0.97, b=1.13, $\Delta=b-a=0.16$. The value of $\Delta$ is obviously less than 0.5, so it is judged that it also has a gray index law with an absolute gray level of 0.16, which is obviously a quasi-exponential law, and a gray prediction model can be established for prediction.

On the basis, according to formula 3.6 we can calculate the smoothness under each value of k as shown in Table 4:

<table>
<thead>
<tr>
<th>k</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth ratio</td>
<td>0.486</td>
<td>0.337</td>
<td>0.276</td>
<td>0.228</td>
<td>0.201</td>
<td>0.174</td>
<td>0.167</td>
<td>0.145</td>
</tr>
</tbody>
</table>

On the basis, calculate the We calculate the value of $\frac{\rho(k+1)}{\rho}$ under different values of k and judge the relationship between it and 1. The calculation results of the value are shown in Table 5:

<table>
<thead>
<tr>
<th>k</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\rho(k+1)}{\rho}$</td>
<td>0.491</td>
<td>0.692</td>
<td>0.819</td>
<td>0.828</td>
<td>0.879</td>
<td>0.869</td>
<td>0.960</td>
<td>0.865</td>
<td>0.0</td>
</tr>
</tbody>
</table>

It can be clearly seen from Table 5 that when k=3...10, the interval of the value of the smooth ratio is $\rho(k) \in [0,0.486]$, combined with the conditions of the smooth ratio discrimination, since $\sigma=0.486<0.5$ is satisfied one of the conditions.

It can be seen from Table 4.3 that when k=2...10, the $\frac{\rho(k+1)}{\rho}$ values of the smooth ratio are all less than 1, which satisfies another condition of the smooth ratio judgment.

4.2. Numerical calculation

According to the calculation process of the gray GM(1,1) model, the predicted value of GM(1,1) is calculated as shown in Table 6:
It can be seen that the relative error value is small, which further shows that the model has a better fitting effect. It can be more intuitively reflected on the curve of the prediction result, as shown in Figure 1:

![Diagram of prediction results](image)

**Figure 1 Diagram of prediction results**

It is not difficult to find that the GM(1,1) model shows excellent prediction accuracy when a large number of samples is not required, and the calculation workload is also small, and the requirements for MATLAB programming are not very high. We can pass the established MATLAB program. come to conclusion. It can be seen from the figure that the demand for

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual value</th>
<th>Predictive value</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>73900</td>
<td>73900</td>
<td>0</td>
</tr>
<tr>
<td>2009</td>
<td>74600</td>
<td>69000</td>
<td>0.075067024</td>
</tr>
<tr>
<td>2010</td>
<td>72200</td>
<td>73200</td>
<td>0.013850416</td>
</tr>
<tr>
<td>2011</td>
<td>74300</td>
<td>77670</td>
<td>0.045356662</td>
</tr>
<tr>
<td>2012</td>
<td>81300</td>
<td>82400</td>
<td>0.013530135</td>
</tr>
<tr>
<td>2013</td>
<td>85900</td>
<td>87420</td>
<td>0.017694994</td>
</tr>
<tr>
<td>2014</td>
<td>92700</td>
<td>92750</td>
<td>0.000539374</td>
</tr>
<tr>
<td>2015</td>
<td>96700</td>
<td>98410</td>
<td>0.017683557</td>
</tr>
<tr>
<td>2016</td>
<td>109000</td>
<td>104410</td>
<td>0.042110092</td>
</tr>
<tr>
<td>2017</td>
<td>110000</td>
<td>110700</td>
<td>0.006363636</td>
</tr>
<tr>
<td>2018</td>
<td>110700</td>
<td>110770</td>
<td>0.006363636</td>
</tr>
</tbody>
</table>
containers is roughly showing an upward trend year by year, and the degree of dispersion between the fitted value of the demand and the actual value is also small, showing a higher prediction accuracy.

5. **Markov prediction**

5.1. **Division status**

Calculate the relative value between the actual value and the predicted value according to the results of the gray prediction model, as the basis for dividing the state, the calculation results are shown in Table 7:

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual value</th>
<th>Predictive value</th>
<th>Relative value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>73900</td>
<td>73900</td>
<td>1.000</td>
</tr>
<tr>
<td>2009</td>
<td>74600</td>
<td>69000</td>
<td>1.081</td>
</tr>
<tr>
<td>2010</td>
<td>72200</td>
<td>73200</td>
<td>0.986</td>
</tr>
<tr>
<td>2011</td>
<td>74300</td>
<td>77670</td>
<td>0.957</td>
</tr>
<tr>
<td>2012</td>
<td>81300</td>
<td>82400</td>
<td>0.987</td>
</tr>
<tr>
<td>2013</td>
<td>85900</td>
<td>87420</td>
<td>0.983</td>
</tr>
<tr>
<td>2014</td>
<td>92700</td>
<td>92750</td>
<td>0.999</td>
</tr>
<tr>
<td>2015</td>
<td>96700</td>
<td>98410</td>
<td>0.983</td>
</tr>
<tr>
<td>2016</td>
<td>109000</td>
<td>104410</td>
<td>1.044</td>
</tr>
<tr>
<td>2017</td>
<td>110000</td>
<td>110700</td>
<td>0.994</td>
</tr>
</tbody>
</table>

According to the relative value in Table 5.1, the relative value status is divided into three groups, as shown in Table 8:

<table>
<thead>
<tr>
<th>Status</th>
<th>Meaning</th>
<th>Interval Range</th>
<th>Year</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>E₂</td>
<td>More accurate</td>
<td>0.998-1.040</td>
<td>2008, 2014, 2016</td>
<td>3</td>
</tr>
<tr>
<td>E₁</td>
<td>underestimate</td>
<td>1.040-1.081</td>
<td>2009</td>
<td>1</td>
</tr>
</tbody>
</table>

5.2. **Test Markovian**

According to the state divided in Table 8, the transition frequency and transition probability matrix are initially constructed:
The Markov property is tested according to the relevant formula, and the results are shown in Table 9:

<table>
<thead>
<tr>
<th>Status</th>
<th>( f_{ij} )</th>
<th>( \ln \left( \frac{P_{ij}}{P_{1}} \right) )</th>
<th>( f_{ij} )</th>
<th>( \ln \left( \frac{P_{ij}}{P_{2}} \right) )</th>
<th>( f_{ij} )</th>
<th>( \ln \left( \frac{P_{ij}}{P_{3}} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>0.575</td>
<td>0</td>
<td>0</td>
<td>0.511</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>0.308</td>
<td>0</td>
<td>0</td>
<td>1.099</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Calculated to obtain \( \chi^2 = 4.986 \), in the case of the selected significance level \( \alpha=0.50 \), \( \chi^2 > \chi^2_{0.50} ((3 - 1)^2) = 3.357 \). Therefore, it has a horse-like character and can be used for Markov prediction.

Since the Markov property has been tested, the two-step and three-step state transition matrices are calculated based on the one-step state transition probability matrix.

\[
P^{(1)} = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.7 & 0 & 0.3 \\ 1 & 0 & 0 \end{bmatrix}
\]

\[
P^{(2)} = \begin{bmatrix} 0.64 & 0.24 & 0.12 \\ 0.72 & 0.28 & 0 \\ 0.6 & 0.4 & 0 \end{bmatrix}
\]

\[
P^{(3)} = \begin{bmatrix} 0.67 & 0.26 & 0.07 \\ 0.63 & 0.29 & 0.08 \\ 0.64 & 0.24 & 0.12 \end{bmatrix}
\]

5.3. **Grey GM(1,1)-Markov prediction model**

Use the 3-step transition probability matrix to predict the container volume in 2018. Select the 3 years closest to the forecast year, and take the corresponding steps to form a new probability matrix, and sum the matrix column vectors. The maximum value is the state of the forecast in 2018, as shown in the Table 11:
It can be seen from the table that the forecast data in 2018 has the greatest probability of being in the state $E_1$. Therefore, it is estimated that the container data in 2018 is in an "overestimated" state, and the state interval is $[0.957, 0.998]$. Therefore, the demand for containers in 2018 is calculated as $\hat{\chi} = 1 / 2 \times (0.957+0.998) \times 110770=108278$. The forecast values from 2008 to 2017 are shown in Table 12:

5.4. Comparison of model accuracy

Using average relative error, posterior difference ratio C, small error probability P and other three indicators, comparing the prediction accuracy of gray GM(1,1) model and gray GM(1,1)-Markov model, the gray GM(1,1)-Markov model's prediction accuracy effect level is higher than gray GM(1,1) model, as shown in Table 13:
This paper draws a scatter plot of the predicted value of the GM(1,1) prediction model and the GM(1,1)-Markov prediction model. It is found that the deviation between the predicted value and the actual value in 2009 and 2016 is large, but the predicted value of the GM(1,1)-Markov prediction model is closer to the actual value in comparison, and in most cases the predicted value of the GM(1,1)-Markov prediction model is higher than that of the GM(1,1) prediction model, so the predicted value in 2011 deviates more from the actual value. It can be seen that the model still has shortcomings in application and needs further optimization.

6. Conclusion

In the process of forecasting the demand for container phases, the GM(1,1) gray forecasting method and the gray-Markov forecasting model have shown good forecasting accuracy. According to the results of the accuracy comparison, it can be known that the two are on average Relative error, posterior error ratio, and small probability error belong to the same level on the three accuracy indicators, but the gray-Markov model reflects a better posterior error ratio. However, in the container forecasting process, there are still many uncertain factors that should be considered. The forecast value still has a large deviation value, which can be solved by a better model or algorithm.

References


